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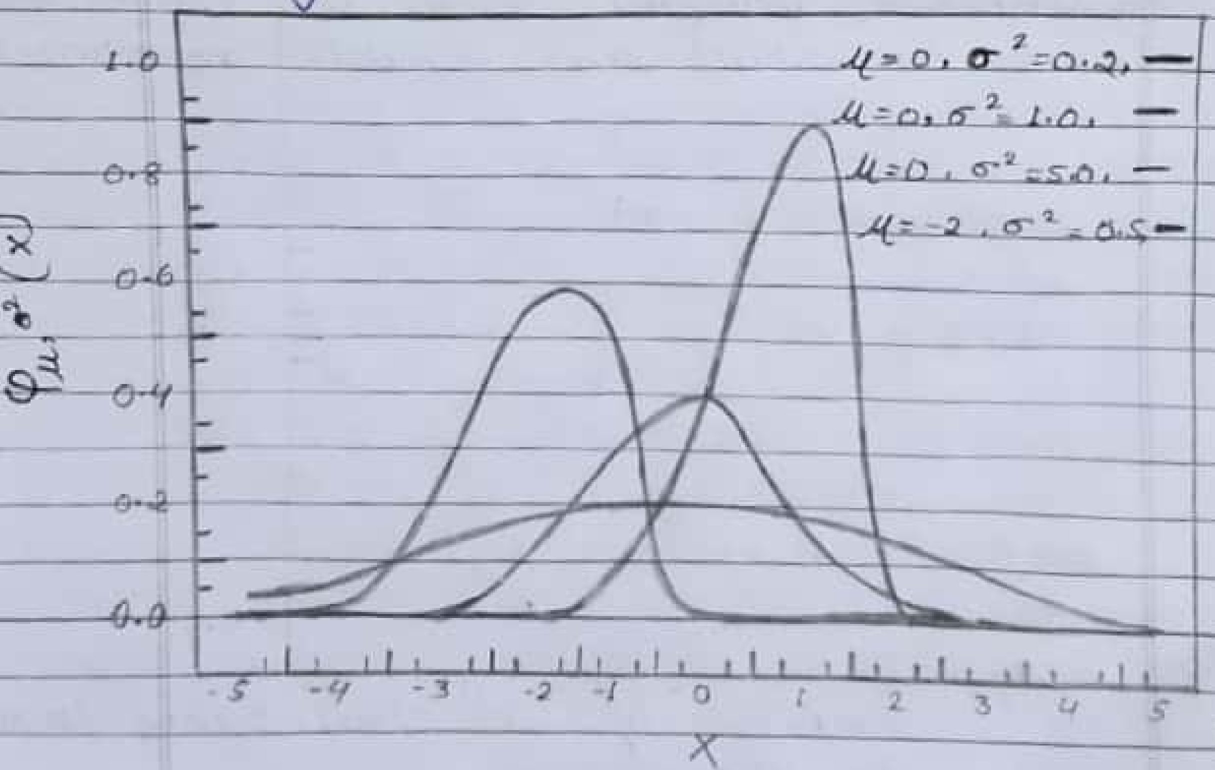
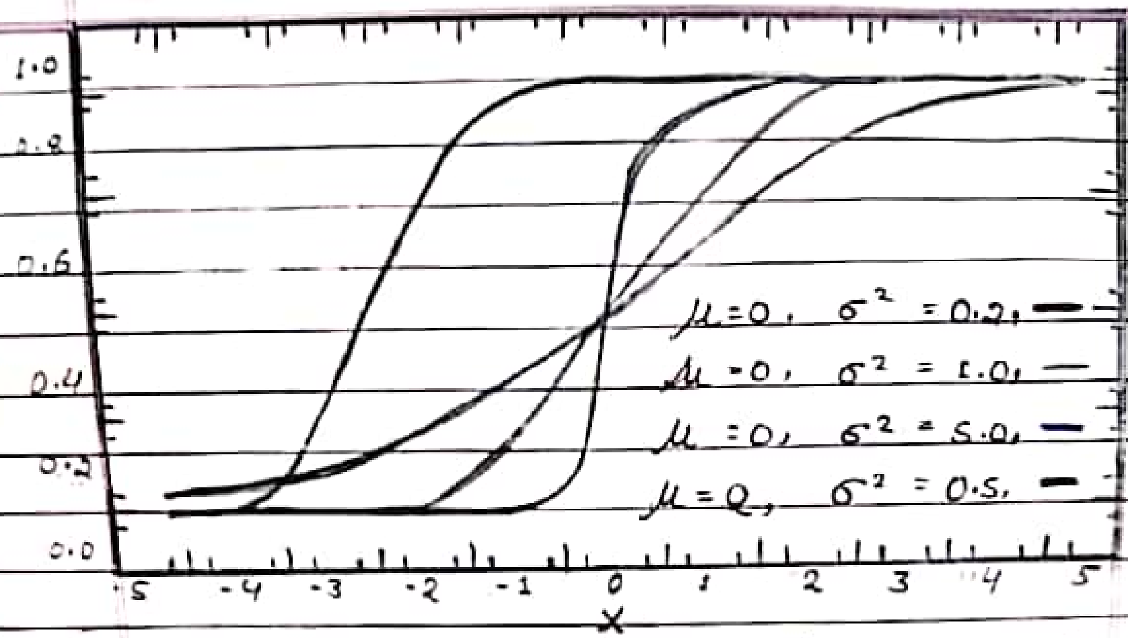
Question :- Write Notes on NORMAL DISTRIBUTION with example ?

Answer :- Normal distribution :-

In probability theory, a normal (or Gaussian or Gauss or laplace-Gauss) distribution is a type of continous probability distribution for a real - valued random variable. The general form of its probability density function is.





Notation	$N(\mu, \sigma^2)$
Parameters	$\mu \in \mathbb{R}$ = mean (location) $\sigma^2 > 0$ = variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
CDF	$\frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$
Quantile	$\mu + \sigma\sqrt{2}\, \operatorname{erf}^{-1}(2p-1)$
Mean	μ
Median	μ
Mode	μ

Variance	σ^2
MAD	$\sigma\sqrt{2/\pi}$
Skewness	0
Ex. kurtosis	0
Entropy	$\frac{1}{2}\log(2\pi e\sigma^2)$
MGF	$\exp(\mu t + \sigma^2 t^2/2)$
CF	$\exp(i\mu t - \sigma^2 t^2/2)$
Fisher information	$I(\mu,\sigma) = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 2/\sigma^2 \end{pmatrix}$
	$I(\mu,\sigma^2) = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 1/2\sigma^2 \end{pmatrix}$
Kullback-Leibler	$D_{KL}(N_0 \parallel N_1) = \frac{1}{2}\left\{\frac{\sigma_0}{\sigma_1}\right\}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Definitions :-

Standard normal distribution :-

The simplest case of a normal distribution is known as the standard normal distribution. This is a special case when $\mu = 0$ and $\sigma = 1$, and it is described by this probability density function:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

Authors differ on which normal distribution should be called the "standard" one. Carl Friedrich Gauss defined the standard normal as having variance $\sigma^2 = 1/2$, that is

$$\varphi(x) = \frac{e^{-x^2}}{\sqrt{\pi}}$$

Cumulative distribution function :-

The cumulative distribution fuction (CDF) of the standard normal distribution, usually denoted with the capital Greek letter Φ (phi), is the integral

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$

The related error function $erf(x)$ gives the probability of a random variable with normal distribution of mean 0 and variance 1/2 falling in the range $[-x, x]$; that is $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$.