

The expression $\frac{d^2y}{dx^2}$ is called the second derivative of the given function $f(x)$.

In the same manner, $\frac{d^3y}{dx^3}$ means that the given function $y = f(x)$ is differentiated thrice. The expression $\frac{d^3y}{dx^3}$ is called the third derivative of the given function and so on.

The other notation for $\frac{d^3y}{dx^3}$ is $f'''(x)$.

Similarly we can determine the fourth - derivative; fifth derivative and in a general way the n^{th} derivative by - differentiating successively the given function $y = f(x)$ four times, five times, ... n times.

Notations: it is convenient to use the following notations for the successive derivatives of a given function $y = f(x)$

first derivative $\frac{dy}{dx}$ by y_1 .

second derivative $\frac{d^2y}{dx^2}$ by y_2 .

Third derivative $\frac{d^3y}{dx^3}$ by y_3 .

n^{th} derivative $\frac{d^ny}{dx^n}$ by y_n .

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08/04/21

Date - 08/04/21

Dept. of Mathematics

D. K. College, Dumraon.

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Class - B.Sc. Part II (Maths. Subsidiary)

Name of the Topic:

Successive Differentiation

Leibnitz theorem

We shall learn the
Successive differentiation of a function

$$y = f(x)$$

Let $y = f(x)$ be any function of x .
We know that if $y = f(x)$ be differentia-
ted once w.r. to the independent variable
 x , then this is denoted by $\frac{dy}{dx}$, the
other notation for $\frac{dy}{dx}$ is $f'(x)$.

The expression $\frac{dy}{dx}$ is called the first
derivative of the function $f(x)$.

Similarly, if $\frac{dy}{dx}$ be differen-
tiated once again, i.e.,

$y = f(x)$ be differentiated twice,
then that is denoted by $\frac{d^2y}{dx^2}$ or $f''(x)$.

Then $(a, b) \in A \times B$

②

$\Rightarrow a \in A$ and $b \in B$.
 $\Rightarrow a \in C$ and $b \in D$. ($\because A \subseteq C$)
 $\Rightarrow (a, b) \in C \times D$

Hence, $A \times B \subseteq C \times D$.

Q3: If $A \subseteq B, C \subseteq D$ Prove that,
 $A \times C \subseteq B \times D$.

Proof:

We have $A \subseteq B$ — (1)

and $C \subseteq D$ — (2)

Let, (x, y) be an arbitrary element
of $A \times C$.

Then $(x, y) \in A \times C$

$\Rightarrow x \in A$ and $y \in C$

$\Rightarrow x \in B$ and $y \in D$ by (1) and (2)

$\Rightarrow (x, y) \in B \times D$.

Hence, by the definition of sets
we have

$A \times C \subseteq B \times D$

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B.Sc.II (Maths Sub)

Date: 08/04/21.

Dept. of Mathematics.

D.K. College, Dumbur.

Class - B.Sc. Part I (Maths Sub.)

Name of the Topic: Set Theory

(Cartesian Product)

Q. (1) Define Cartesian product of two sets.

Ans: The Cartesian product of two sets A and B is the set of all distinct ordered pairs (a,b) where first co-ordinate $a \in A$ and the second co-ordinate $b \in B$. The Cartesian product of A and B is denoted by $A \times B$. So by definition

$$A \times B = \{ (a,b) / a \in A \text{ and } b \in B \}$$

for example: Let $A = \{ (1,2) \}$

$$B = \{ -1, 0, -2 \}$$

$$A \times B = \{ (1,-1), (1,0), (1,-2), (2,-1), (2,0), (2,-2) \}$$

Q. (2) if A, B and C be given sets and $A \subseteq C$ then prove that $A \times B \subseteq C \times B$.

Sol: Let, (a,b) be an arbitrary element of $A \times B$

$A \times B$

[P.T.O.]

(A) Find the differential equation of all circles passing through the origin and having their centre on the axis of x .

Solⁿ:

The eqⁿ of circles passing through the origin and having its centre on the axis of x is given by $x^2 + y^2 + 2gx = 0$ — (1)

Here g is a parameter (i.e. arbitrary constant)

\therefore The eqⁿ of circles

$$x^2 + y^2 + 2gx = 0 \quad \text{--- (1)}$$

differentiating (1) with respect to 'x',

we get, $2x + 2yy' + 2g = 0$.

$$g = -(x + yy')$$

Substituting the value of g in (1),

we get, $x^2 + y^2 - 2(x + yy')x = 0$

$$\text{Or, } x^2 + y^2 - 2x^2 - 2xyy' = 0$$

$$\text{Or, } 2xyy' = y^2 - x^2$$

This is the required differential eqⁿ

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⑤ Construct a differential eqn by the elimination of the arbitrary constants a, b and c from the eqn

$$y = ae^{2x} + be^{-3x} + ce^x.$$

Sol.ⁿ - we have,

$$y = ae^{2x} + be^{-3x} + ce^x \quad \text{--- (1)}$$

differentiating (1) w.r. to 'x',

we get,

$$y_1 = 2ae^{2x} - 3be^{-3x} + ce^x \quad \text{--- (2)}$$

Again differentiating (2) with respect to 'x'

we get,

$$y_2 = 4ae^{2x} + 9be^{-3x} + ce^x \quad \text{--- (3)}$$

differentiating (3) with respect to 'x',

we get,

$$y_3 = 8ae^{2x} - 27be^{-3x} + ce^x \quad \text{--- (4)}$$

Multiplying (2) by 7, we get,

$$7y_1 = 14ae^{2x} - 21be^{-3x} + 7ce^x \quad \text{--- (5)}$$

Subtracting (5) from (4),

we get,

$$y_3 - 7y_1 = -6ae^{2x} - 6be^{-3x} - 6ce^x$$

$$= -6(ae^{2x} + be^{-3x} + ce^x)$$

$$= -6y \quad \text{by (1)}$$

Hence the required differential equation is $y_3 - 7y_1 + 6y = 0$

(2) Form the differential equation of the family of curves

$y = A \sin(mx+B)$, where A and B are arbitrary constant.

Solⁿ:-

$$y = A \sin(mx+B) \quad \text{--- (1)}$$

Differentiating (1) with respect to 'x', we get

$$y' = Am \cos(mx+B) \quad \text{--- (2)}$$

Differentiating (2) with respect to 'x',

we get,

$$y'' = -Am^2 \sin(mx+B)$$

$$\text{Or, } y'' = -m^2 y, \text{ by virtue of (1)}$$

$$\text{Or, } y'' + m^2 y = 0 \text{ which is}$$

free from A and B .

This is the required differential equation.

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Dept of Mathematics

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Q. No. II (Maths)

Class: B. Sc. Part II (Maths. Hours)

Name of the topic: Formation of diff
differential equation by Elimination.

~~Q. No. I~~ Find the differential equation
corresponding to the family of
curves $xy^2 = (x-c)^2$ where c is an
arbitrary constant.

Soln: We have $xy^2 = (x-c)^2$ (1)

Differentiating (1) w.r to x :

We get, $2xy \frac{dy}{dx} = 2(x-c)$

Or, $8a^3y^3 \left(\frac{dy}{dx}\right)^3 = 27 \{(x-y)^3\}^2$
 $= 27 \cdot a^2y^4 \cdot \text{by interchanging}$

Or, $8a \left(\frac{dy}{dx}\right)^3 = 27y$ which is

free from the arbitrary constant.

This is the required differential
equation.