(iii) Transpose Matrix :- A matrix where rows are identical with the corresponding columns of a matrix A is called the transpose of A and is denoted by A'. If A is an m×n matrix then A' will be n×m matrix. For example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ then } A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

(iv) Diagonal Matrix :- A Square matrix whose all elements except the leading diagonal elements are zero, is called a diagonal matrix it is denoted by $[d_i]$. for example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ is diagonal matrix}$$

(v) identity matrix or Unit matrix :- A Square matrix whose elements in the leading diagonal are all 1 and rest are all 0 is called Unit matrix. for example.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is a } 3\times3 \text{ Unit matrix}$$

09.04.21

B.Sc. I (Maths Subs)

Dept of Mathematics.
D. K. College, Dumraon.
Dr. Birendra Prasad.
Asst. Prof. (Guest faculty)

Class. B. Sc. Part I (Maths Subs)
Name of the Topic: Matrices.

Defn

(I) Matrix:- A set of mn numbers or elements of a field arranged in any array of m rows and n columns

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is called a matrix. a_{ij}'s are called elements of the matrix.
if $m \neq n$, then the matrix is rectangular.
if $m = n$, the matrix is a square matrix.
if $m = 1$, the matrix is a row matrix.
if $n = 1$, the matrix is a column matrix

(II) Singular matrix:- A square matrix A is said to be singular if the determ $|A| = 0$
if $|A| \neq 0$, it is non-singular.

Thm. $(A\times B)\cap(C\times D) = (A\cap C)\times(B\cap D)$ ②
Where A, B, C and D are sets.

Proof: Let (x,y) be an arbitrary element of $(A\times B)\cap(C\times D)$

$\therefore (x,y) \in (A\times B)\cap(C\times D)$
$\Rightarrow (x,y)\in(A\times B) \wedge (x,y)\in(C\times D)$
$\Rightarrow x\in A \wedge y\in B \wedge (x\in C \wedge y\in D)$
$\Rightarrow (x\in A \wedge x\in C)\wedge(y\in B\wedge y\in D)$
$\Rightarrow x\in(A\cap C)\ y\in(B\cap D)$
$\Rightarrow (x,y)\in(A\cap C)\times(B\cap D)$

Therefore, by definition of subset,
$(A\times B)\cap(C\times D)\subseteq(A\cap C)\times(B\cap D)$ — (1)

Again,
Let, $(a,b)\in(A\cap C)\times(B\cap D)$ be arbitrary

$\therefore (a,b)\in(A\cap C)\times(B\cap D)$
$\Rightarrow a\in(A\cap C)\wedge b\in(B\cap D)$
$\Rightarrow a\in A\wedge a\in C\wedge(b\in B\wedge b\in D)$
$\Rightarrow (a\in A\wedge b\in B)\wedge(a\in C\wedge b\in D)$
$\Rightarrow (a,b)\in(A\times B)\wedge(a,b)\in(C\times D)$
$\Rightarrow (a,b)\in(A\times B)\cap(C\times D)$

Therefore, by definition of subset,
$(A\cap C)\times(B\cap D)\subseteq(A\times B)\cap(C\times D)$ — (2)

Now, from (1) and (2), we have,
$(A\times B)\cap(C\times D) = (A\cap C)\times(B\cap D)$

05.04.21

again, Let (a,b) be an arbitrary element of $(A\times B)\cup(A\times C)$ —— ①

$\therefore (a,b)\in(A\times B)\cup(A\times C)$

$\Rightarrow (a,b)\in(A\times B)$ or $(a,b)\in(A\times C)$

$\Rightarrow (a\in A$ and $b\in B)$ or $(a\in A$ and $b\in C)$

$\Rightarrow a\in A$ and $(b\in B$ or $b\in C)$

$\Rightarrow a\in A$ and $b\in(B\cup C)$

$\Rightarrow (a,b)\in A\times(B\cup C)$

Hence, by definition of subset,

$(A\times B)\cup(A\times C)\subseteq A\times(B\cup C)$ —— ②

Thus, from ① and ②, we find

$$A\times(B\cup C)=(A\times B)\cup(A\times C)$$

Proof of (ii)

Let $(a,b)\in A\times(B\cap C)$ be an arbitrary,

then, $(a,b)\in A\times(B\cap C)$

$\Rightarrow a\in A\wedge b\in(B\cap C)$

$\Rightarrow a\in A\wedge(b\in B\wedge b\in C)$

$\Rightarrow (a\in A\wedge b\in B)\wedge(a\in A\wedge b\in C)$

$\Rightarrow (a,b)\in(A\times B)\wedge(a,b)\in(A\times C)$

$\Rightarrow (a,b)\in(A\times B)\cap(A\times C)$

Hence by definition of ~~set~~ subset

$A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$ —— ①

Again, Let $(c,d)\in(A\times B)\cap(A\times C)$ be arbitrary

then, $(c,d)\in(A\times B)\cap(A\times C)$

$\Rightarrow (c,d)\in A\times B\wedge(c,d)\in(A\times C)$

$\Rightarrow (c\in A\wedge d\in B)\wedge(c\in A\wedge d\in C)$

$\Rightarrow c\in A\wedge(d\in B\wedge d\in C)$

$\Rightarrow c\in A\wedge d\in(B\cap C)$

$\Rightarrow (c,d)\in A\times(B\cap C)$

Hence, by definition of subset,

$(A\times B)\cap(A\times C)\subseteq A\times(B\cap C)$ —— ②

Now from ① and ② we have, $A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$

Date: 09.04.21.

B.Sc. Part II
Maths Hons

Dept. of Mathematics.
D.K. College, Dumraon.
Dr. Birendra Prasad,
Asst. Prof.
(Guest faculty)

Class.- B.Sc. Part II (Maths. Hons)
Name of the Topic:- Differential equation of the first order and first degree.

(1) Solve $(1+y^2)dx + (1+x^2)dy = 0$.

Soln:- From the given eqn, we have

$$(1+x^2)dy = -(1+y^2)dx$$

$$\Rightarrow \frac{dy}{1+y^2} = -\frac{dx}{1+x^2}$$

Integrating, we get

$$\int \frac{dy}{1+y^2} = -\int \frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1}y = -\tan^{-1}x + K$$

$$\Rightarrow \tan^{-1}y + \tan^{-1}x = \tan^{-1}c \quad (\because K = \tan^{-1}c \text{ writing})$$

$$\Rightarrow \tan^{-1}\frac{y+x}{1-yx} = \tan^{-1}c$$

$$\Rightarrow \frac{y+x}{1-yx} = c$$

$$\Rightarrow x+y = c(1-xy)$$

Which is the reqd. solution.

(2) Solve $\frac{dy}{dx} = e^{x+y} + x^2 e^y$.

Soln:- The given eqn can be written as

$$\frac{dy}{dx} = e^x . e^y + x^2 e^y$$

$$= (e^x + x^2) e^y.$$

$$\Rightarrow \frac{dy}{e^y} = (e^x + x^2) dx$$

$$\Rightarrow e^{-y} dy = (e^x + x^2) dx.$$

Hence, Integrating, we have

$$\int e^{-y} dy = \int (e^x + x^2) dx$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^x + \frac{x^3}{3} + c$$ where c is a constant

$$\Rightarrow e^x + \frac{x^3}{3} + e^{-y} = k.$$

Which is the required solution

(3) Solve $(e^y + 1) \cos x dx + e^y \sin x dy = 0$

Soln:- From the given eqn, we have

$$\frac{\cos x}{\sin x} dx + \frac{e^y}{e^y + 1} dy = 0$$

$$\Rightarrow \int \frac{\cos x}{\sin x} dx + \int \frac{e^y}{e^y + 1} dy = 0$$

$$\Rightarrow \log \sin x + \log (e^y + 1) = \log C$$

$$\Rightarrow \log \{\sin x (e^y + 1)\} = \log C$$

$$\therefore \sin x (e^y + 1) = C.$$

(4.) Solve $y\,dx - x\,dy = xy\,dx$.

Sol^n:- from the given equation, we have

$$x\,dy = (y - xy)\,dx = y(1-x)\,dx$$

$$\Rightarrow \frac{dy}{y} = \frac{1-x}{x}dx = \left(\frac{1}{x}-1\right)dx.$$

Integrating, we get

$$\int \frac{dy}{y} = \int\left(\frac{1}{x}-1\right)dx.$$

$$\Rightarrow \log y = \log x - x + c$$

$$\Rightarrow \log y - \log x = c - x$$

$$\Rightarrow \log\frac{y}{x} = c - x$$

$$\Rightarrow \frac{y}{x} = e^{c-x}$$

$$\Rightarrow y = x e^{c-x}$$

which is the reqd sol^n.

(5) Solve $x(y^2+1)dx + y(x^2+1)dy = 0$,

Sol^n:- The given eqn can be written as

$$y(x^2+1)dy = -x(y^2+1)dx$$

$$\Rightarrow \frac{y\,dy}{y^2+1} = \frac{-x\,dx}{x^2+1}$$

Integrating, we get

$$\int\frac{y\,dy}{y^2+1} = -\int\frac{x\,dx}{x^2+1}$$

$$\Rightarrow \frac{1}{2}\int\frac{2y\,dy}{y^2+1} = -\frac{1}{2}\int\frac{2x\,dx}{x^2+1}$$

$$\Rightarrow \frac{1}{2}\log(y^2+1) = -\frac{1}{2}\log(x^2+1) + c$$

$$\Rightarrow \frac{1}{2}\log(y^2+1) + \frac{1}{2}\log(x^2+1) = c$$

$$\Rightarrow \log\{(y^2+1)(x^2+1)\} = 2c = \log k \text{ (say)}$$

$$\Rightarrow (y^2+1)(x^2+1) = k.$$

which is the reqd. solution.

19.04.21

Dept of Mathematics.
D.K. College, Dumraon
Dr. Birendra Prasad,
Asst. Prof. (Guest Faculty)

B.Sc. Part I (Maths Hons)

Name of the Topic Cartesian Product of Sets.

Thm:- if A, B, C be the three sets, then (i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

(ii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

Proof of (i)

Let (x,y) be an arbitrary elements of $A\times(B\cup C)$

$\therefore (x,y) \in A\times(B\cup C)$

$\Rightarrow x\in A$ and $y\in(B\cup C)$

$\Rightarrow x\in A$ and $(y\in B$ or $y\in C)$

$\Rightarrow (x\in A$ and $y\in B)$ or $(x\in A$ and $y\in C)$

$\Rightarrow (x,y)\in(A\times B)$ or $(x,y)\in A\times C$

$\Rightarrow (x,y)\in(A\times B)\cup(A\times C)$

Hence by the definition of Subset

$A\times(B\cup C)\subseteq(A\times B)\cup(A\times C)$ ——— (1)