We want to prove that the representation $A = P + Q$ is unique.

If possible, let $A = R + S$ be any other representation where R is symmetric and S is skew-symmetric.

$\therefore R' = R$ and $S' = -S$

$$\Rightarrow A' = (R+S)' = R' + S' = R - S.$$

$$\therefore A + A' = R + \cancel{S} + R - \cancel{S} = 2R$$

$$A - A' = R + S - (R - S)$$
$$= \cancel{R} + S - \cancel{R} + S = 2S.$$

$$\Rightarrow R = \frac{1}{2}(A + A') = P$$

and $S = \frac{1}{2}(A - A') = Q$

$\therefore A = P + Q$ is unique representation.

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Question

Show that every square matrix can be uniquely expressed as the sum of a symmetric and skew-symmetric matrix.

Ans. Let A be a square matrix. Then

$$A = \frac{1}{2}A + \frac{1}{2}A = \frac{1}{2}A + \frac{1}{2}A' + \frac{1}{2}A - \frac{1}{2}A'$$

$$\Rightarrow A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$= P + Q \text{ (say)}$$

where $P = \frac{1}{2}(A+A')$

$Q = \frac{1}{2}(A-A')$

We note that $P' = \left(\frac{1}{2}(A+A')\right)' = \frac{1}{2}(A+A')'$

$$= \frac{1}{2}(A'+A) = \frac{1}{2}(A+A') = P.$$

and $Q' = \left(\frac{1}{2}(A-A')\right)' = \frac{1}{2}(A-A')'$

$$= \frac{1}{2}(A'-A)$$

$$= -\frac{1}{2}(A-A')$$

$$= -Q.$$

$\therefore$ P is symmetric and Q is skew symmetric.

~~Hence $A = P + Q$.~~

(2) Inverse Matrix:-

If the matrix A and B are correlated that $AB = BA = I$, the unity matrix, then B is called the inverse of A and is written as A^{-1}.

(3) Reciprocal Matrix:- If A is a non-singular matrix, the matrix $\frac{adj.A}{|A|}$ is ~~called the reciprocal of A.~~

(4) Orthogonal Matrix:- A Square matrix A is said to be Orthogonal matrix if $A'A = I$

A' and I, being the transpose of A and Unit matrix respectively.

Q:- Define Hermitian matrix and Skew-Hermitian matrix.

Ans: Hermitian matrix: A Square matrix $A = [a_{ij}]_{n \times n}$ is said to be Hermitian

$$a_{ij} = \overline{a_{ji}}$$

i.e. iff $(\bar{A})' = A$.

i.e iff $A^{\theta} = A$.

It may be noted each diagonal entry of a Hermitian matrix is purely real.

Skew-Hermitian matrix: A Square matrix $A = [a_{ij}]_{n \times n}$ is Skew-Hermitian iff $a_{ij} = -\overline{a_{ji}}$, i.e, iff $(\bar{A})' = -A$, i.e iff

It may be noted that each diagonal entry of a Skew-Hermitian matrix is 0 or purely imaginary.

Dept. of Mathematics.
Class. B.Sc. Part I (Maths Hons)
Paper - I.

Name of the Topic: Matrices.

Def^n:

(1) <u>Symmetric and Skew-Symmetric Matrix</u>: A Square matrix A is called a Symmetric matrix if it is equal to its transpose A' i.e A = A'.

if $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then $A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

i.e $A = A'$.

∴ Here A is a Symmetric matrix.

if for Square matrix A, we get $A = -A'$, then A is called a Skew Symmetric matrix.

$A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$ then $A' = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$

Clearly $A = -A'$.

(2) <u>Adjoint Matrix</u>:- if A is a Square matrix and B is a matrix whose elements are the Cofactors of the corresponding elements in |A| then the transpose of B is called the adjoint or adjugate matrix of A.

(4) Integrate $\frac{dy}{dx} = \sec(x+y)$

Solⁿ: we have $\frac{dy}{dx} = \sec(x+y)$ ——— (i)

Put $x+y = v$

D.B.S. w. r. to x: we get,

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\text{or, } \frac{dy}{dx} = \frac{dv}{dx} - 1$$

Substituting these values in (i) we get

$$\frac{dv}{dx} - 1 = \sec v$$

$$\text{or, } \frac{dv}{dx} = 1 + \sec v$$

$$\text{or, } dx = \frac{dv}{1+\sec v} = \frac{dv}{1+\frac{1}{\cos v}} = \frac{\cos v\, dv}{1+\cos v}$$

$$= \frac{(1+\cos v)-1}{1+\cos v}dv = \frac{1+\cos v}{1+\cos v}dv - \frac{1}{1+\cos v}dv$$

$$= dv - \frac{1}{2\cos^2\frac{v}{2}}dv = dv - \frac{1}{2}\sec^2\frac{v}{2}\,dv$$

Integrating, we get

$$x = v - \tan\frac{v}{2} + c$$

Where c is the constant of integ

$$\text{or, } x = x+y - \tan\frac{x+y}{2} + c$$

$$\text{or, } \tan\frac{x+y}{2} = y + c. \quad \text{Ans}$$

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(3) Solve
$$\frac{x\,dx+y\,dy}{x\,dy-y\,dx}=\sqrt{\left(\frac{a^2-x^2-y^2}{x^2+y^2}\right)}$$

Solⁿ:- We have,
$$\frac{x\,dx+y\,dy}{x\,dy-y\,dx}=\sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}} \quad \text{--- (1)}$$

Put $x=r\cos\theta$ and $y=r\sin\theta$.
$\therefore x^2+y^2=r^2(\cos^2\theta+\sin^2\theta)=r^2\cdot 1=r^2$
Differentiating, we get
$2x\,dx+2y\,dy=2r\,dr$
Or, $x\,dx+y\,dy=r\,dr$
Again, $\frac{y}{x}=\frac{r\sin\theta}{r\cos\theta}=\tan\theta$.
Or, $\theta=\tan^{-1}\frac{y}{x}$
Differentiating, we get
$$d\theta=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot d\left(\frac{y}{x}\right)=\frac{x^2}{x^2+y^2}\cdot\frac{x\,dy-y\,dx}{x^2}$$
$$=\frac{x\,dy-y\,dx}{r^2}$$
Or, $x\,dy-y\,dx=r^2\,d\theta$
Substituting these values in (1), we get
$$\frac{r\,dr}{r^2\,d\theta}=\sqrt{\left(\frac{a^2-r^2}{r^2}\right)}=\frac{\sqrt{a^2-r^2}}{r}$$
Or, $d\theta=\frac{dr}{\sqrt{a^2-r^2}}$
Integrating both sides, we get
$\theta+c=\sin^{-1}\frac{r}{a}$ where c is the const.
Or, $r=a\sin(\theta+c)$
Or, $\sqrt{x^2+y^2}=a\sin\left(c+\tan^{-1}\frac{y}{x}\right)$ Ans

Integrate $(x+y+1)\frac{dy}{dx} = 1$.

Soln:- We have,

$$(x+y+1)\frac{dy}{dx} = 1 \quad \text{--- (1)}$$

Put $x+y+1 = t$.

D. B. S. W. r. to x:

we get, $1 + \frac{dy}{dx} = \frac{dt}{dx}$

or, $\frac{dy}{dx} = \frac{dt}{dx} - 1$

Substituting these values in (1), we get,

$$t\left(\frac{dt}{dx} - 1\right) = 1.$$

or, $t\frac{dt}{dx} - t = 1.$

or, $t\frac{dt}{dx} = 1+t$

or, $dx = \frac{t}{1+t}\,dt.$

or, $dx = \frac{(1+t)-1}{1+t}\,dt$

$= \frac{1+t}{1+t}\,dt - \frac{1}{1+t}\,dt.$

or, $dx = dt - \frac{1}{1+t}\,dt$

Integrating, we get $x = t - \log(1+t) + \log$

where K is the const. of In

or, $x = x+y+1 - \log(1+x+y+1) + \log$

or, $\log\frac{x+y+2}{C} = y$, where $1+\log k$

or, $x+y+2 = Ce^{y}.$

Date: 12/04/21. (1)

Dept. of Maths.
B.Sc. II (Maths Hons.)
Paper. IV

Name of the Topic: Diff. equation.
Form: Reducible into variable separable

Solve $\frac{dy}{dx} = (x+y)^2$

Soln: We have $\frac{dy}{dx} = (x+y)^2$ ——— (1')

Put $x+y = z$

D.B.S.W.r. to 'x';

We get, $1 + \frac{dy}{dx} = \frac{dz}{dx}$

Or, $\frac{dy}{dx} = \frac{dz}{dx} - 1$

Substituting these values in (1'), we get

$\frac{dz}{dx} - 1 = z^2$

Or, $\frac{dz}{dx} = 1 + z^2$

Or, $dx = \frac{dz}{1+z^2}$

Integrating, we get $c + x = \tan^{-1} z$

where c is the const

Or, $z = \tan(c+x)$

Or, $x+y = \tan c+x$