The expression $\frac{d^2y}{dx^2}$ is called the second derivative of the given function $f(x)$.
In the same manner, $\frac{d^3y}{dx^3}$ means that the given function $y = f(x)$ is differentiated thrice. The expression $\frac{d^3y}{dx^3}$ is called the third derivative of the given function and so on.
The other notion for $\frac{d^3y}{dx^3}$ is $f'''(x)$

Similarly we can determine the fourth derivative, fifth derivative and in a general way the n^{th} derivative by differentiating successively the given function $y = f(x)$ four times, five times, ---- n times.

Notations: it is convenient to use the following notations for the successive derivative of a given function $y = f(x)$

first derivative $\frac{dy}{dx}$ by y_1.

second derivative $\frac{d^2y}{dx^2}$ by y_2.

Third derivative $\frac{d^3y}{dx^3}$ by y_3.

— — — — — — — — —

n^{th} derivative $\frac{d^ny}{dx^n}$ by y_n.

08/04/21

Date - 08/04/21

Dept. of Mathematics.
D. K. College, Dumraon.
Dr. Birendra Prasad.
Asst. Prof. (Guest Faculty)
Class. B. Sc. Part II (Maths. Subsidiary)

Name of the Topic:

Successive Differentiation
&
Leibnitz theorem

We shall learn the Successive differentiation of a function $y=f(x)$

Let $y=f(x)$ be any function of x. We know that if $y=f(x)$ be differentiated once w.r. to the independent variable x, then this is denoted by $\frac{dy}{dx}$, The other notion for $\frac{dy}{dx}$ is $f'(x)$.

The expression $\frac{dy}{dx}$ is called the first derivative of the function $f(x)$.

Similarly, if $\frac{dy}{dx}$ be differentiated once again, i.e,

$y=f(x)$ be differentiated twice, then that is denoted by $\frac{d^2y}{dx^2}$ or $f''(x)$.

②

Then $(a,b) \in A \times B$

$\Rightarrow a \in A$ and $b \in B$.

$\Rightarrow a \in C$ and $b \in B$. ($\because A \subseteq C$)

$\Rightarrow (a,b) \in C \times B$

Hence, $A \times B \subseteq C \times B$.

Q③:- If $A \subseteq B$, $C \subseteq D$ Prove that.

$A \times C \subseteq B \times D$.

Proof:- We have $A \subseteq B$ ——— (1)

and $C \subseteq D$ ——— (2)

Let, (x,y) be an arbitrary element of $A \times C$.

Then $(x,y) \in A \times C$

$\Rightarrow x \in A$ and $y \in C$

$\Rightarrow x \in B$ and $y \in D$ by (1) and (

$\Rightarrow (x,y) \in B \times D$.

Hence, by the definition of Subs we have

$A \times C \subseteq B \times D$

08/4/21

B.Sc. I (Maths Subs)

① Date: 08/04/21.

Dept. of Mathematics.
D.K. College, Dumraon.
Class - B.Sc. Part I (Maths Subs.)
Name of the Topic: Set Theory:
(Cartesian Product)

Q. (1) Define Cartesian Product of two sets.

Ans: The cartesian product of two sets A and B is the set of all distinct ordered pairs (a, b) where first co-ordinate $a \in A$ and the second co-ordinate $b \in B$. The cartesian product of A and B is denoted by $A \times B$. So by definition

$$A \times B = \{(a, b) / a \in A \text{ and } b \in B\}$$

For example: Let $A = \{1, 2\}$

$B = \{-1, 0, -2\}$

$$A \times B = \{(1,-1), (1,0), (1,-2), (2,-1), (2,0), (2,-2)\}$$

Q. (2) if A, B and C be given sets and $A \subseteq C$ then prove that $A \times B \subseteq C \times B$.

Sol^n: Let, (a, b) be an arbitrary element of $A \times B$

[P.T.O.]

(4) Find the differential equation of all circles passing through the origin and having their centres on the axis of x.

Soln:

The eqn of circles passing through the origin and having its centre on the axis of x is given by $x^2+y^2+2gx=0$ — (1)

Here g is a parameter (i.e. arbitrary constant)

∵ The eqn of circles

$$x^2+y^2+2gx=0 \quad \text{— (1)}$$

differentiating (1) with respect to 'x',

we get, $2x+2yy'+2g=0$.

$$g = -(x+yy')$$

Substituting the value of g in (1),

we get, $x^2+y^2-2(x+yy')x=0$

Or, $x^2+y^2-2x^2-2xyy'=0$

Or, $2xyy' = y^2-x^2$

This is the required differential eqn

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(5) Construct a differential eqn by the elimination of the arbitrary constants a, b and c from the eqn

$$y = ae^{2x} + be^{-3x} + ce^{x}.$$

Soln:- We have,

$$y = ae^{2x} + be^{-3x} + ce^{x} \quad \text{--- (1)}$$

differentiating (1) w.r. to 'x',
we get,

$$y_1 = 2ae^{2x} - 3be^{-3x} + ce^{x} \quad \text{--- (2)}$$

Again differentiating (2) with respect to x
we get,

$$y_2 = 4ae^{2x} + 9be^{-3x} + ce^{x} \quad \text{--- (3)}$$

differentiating (3) with respect to 'x',
we get,

$$y_3 = 8ae^{2x} - 27be^{-3x} + ce^{x} \quad \text{--- (}$$

Multiplying (2) by 7, we get,

$$7y_1 = 14ae^{2x} - 21be^{-3x} + 7ce^{x}$$

Substracting (5) from (4),
we get,

$$y_3 - 7y_1 = -6ae^{2x} - 6be^{-2x} - 6ce^{x}$$

$$= 6(ae^{2x} + be^{-3x} + ce^{x})$$

$$= -6y \quad \text{by (1)}$$

Hence the required differential equation is $\boxed{y_3 - 7y_1 + 6y = 0}$

(15)

(2) Form the differential equation of the family of curves

$y = A\sin(mx+B)$, where A and B are orbitrary constant.

Sol^n:- $\because y = A\sin(mx+B)$ —— (1)

Differentiating (1) with respect to 'x', we get

$y' = Am\cos(mx+B)$ —— (2)

Differentiating (2) with respect to 'x', we get,

$y'' = -Am^2\sin(mx+B)$

Or, $y'' = -m^2 y$, by virtue of (1)

Or, $y'' + m^2 y = 0$ which is free from A and B.

This is the required differential equation.

B.Sc. II (Maths Hons)

Date: 08/4/21.

Dept of Mathematics
D.K. College, Dumraon.
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Class: B.Sc. Part II (Maths. Hons.)

Name of the Topic: Formation of differential equation by Elimination.

Q. (1) Find the differential equation corresponding to the family of curves $ay^2 = (x-c)^3$ where c is an orbitrary constant.

Solⁿ: We have $ay^2 = (x-c)^3$ ——— (1)

differentiating (1) w.r. to x:

we get, $2ay\frac{dy}{dx} = 3(x-c)^2$

Or, $8a^3y^3\left(\frac{dy}{dx}\right)^3 = 27\{(x-c)^3\}^2$

$= 27.a^2y^4$, by virtue of (1)

Or, $8a\left(\frac{dy}{dx}\right)^3 = 27y$ which is free from the orbitrary constant c.

This is the required differential equation.