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Dept. of Mathe.

B.Sc. Part II

(Mathe. Honors)

Paper - III

Real Analysis.

Name of the Topic:

DEDKIND'S THEORY.

Dedkind's Cut: A Dedkind's cut is an

ordered pair (A_1, A_2) of sets of rational number having the following properties:

[D₁]: $A_1, A_2 \neq \emptyset$

[D₂]: $A_1 \cup A_2 = \mathbb{Q}$

[D₃]: $A_1 < A_2$

[D₄] A_1 does not possess a greatest rational number.

Sum of two cuts:

Let $\alpha = (A_1, A_2)$, $\beta = (B_1, B_2)$,

We construct following two classes:

- (i) The class C_1 consisting of all rational numbers of the form $x_1 + y_1$ where x_1 denotes any number of A_1 and y_1 any number of B_1 .

(ii) The class C_1 consisting of all other rational numbers.

Here, we will verify the ordered

pair $\gamma = (C_1, C_2)$ is a cut.

(i) Clearly $\gamma = (C_1, C_2) \neq \emptyset$.

(ii) Let $u \in A_2$ and $v \in B_2$,
 u and v are rational.

then $u > x_1$ and $v > y_1$ so that

$u+v > x_1+y_1$ if $x_1 \in A_1$ and $y_1 \in B_1$,

$\Rightarrow u+v \in C_2$. So C_1 does not contain every rational.

(iii) Let $u \in C_1$ and $t < u$

$\Rightarrow u = p+q$ for some $p \in A_1$ and $q \in B_1$.

Let us choose a rational k such that

$t = k+q$. then $t < u \Rightarrow k < p$.

then $k \in A_1 \Rightarrow t \in A_1$

(iv) Let $r \in C_1$,

then $r = p+q$ for some $p \in A_1$ and $q \in B_1$.

Since (A_1, A_2) is a cut then there is rational $u > p$ such that $u \in A_1$.

Then $u+q \in C_1$. Since $u \in A_1$ and $q \in B_1$,

also, $u+q > p+q = r$. So that r is not the largest rational in C_1 .

Here C_1 does not contain the largest rational. So all the three conditions are satisfied and hence $\gamma = (C_1, C_2)$ is a cut. This cut is called the

(3)

Sum of two cuts, i.e.,

$$(A_1, A_2) + (B_1, B_2)$$

$$\therefore (C_1, C_2) = (A_1, A_2) + (B_1, B_2)$$

$$\text{i.e., } \gamma = \alpha + \beta$$

Product of Two cuts:

Let us suppose (A_1, A_2) and (B_1, B_2) be two non negative cuts. Let (C_1, C_2) be product of two cuts (A_1, A_2) and (B_1, B_2) . Then we form the class C_1 consisting of all rational numbers of the form $x_1 y_1$, where $x_1 \in A_1$ and $y_1 \in B_1$ and the class C_2 consisting of all other rational numbers. We can write $(C_1, C_2) = (A_1, A_2)(B_1, B_2)$

if (A_1, A_2) is negative and (B_1, B_2) is non-negative. then their product is defined as

$$(A_1, A_2)(B_1, B_2) = - \left[\left\{ - (A_1, A_2) \right\} (B_1, B_2) \right]$$

if (A_1, A_2) and (B_1, B_2) are both negative

then we define their product as

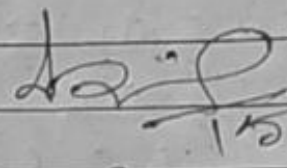
$$(A_1, A_2)(B_1, B_2) = \left[- (A_1, A_2) \right] \left[- (B_1, B_2) \right]$$

(4)

The Unit Cut:

if $I_1 =$ The set of all rational numbers < 1 and I_2 consists of all other rational numbers. Then $I = (I_1, I_2)$ is called the unit cut.

In other words, the cut corresponding to the rational number 1 is called the unit cut.



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