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Dept. Of Maths.
B.Sc. Part II
(Maths. Hons.)
Paper - III
Real Analysis.

Name of the Topic:
DEDKIND'S THEORY.

Dedkind's Cut: A Dedkind's Cut is an Ordered pair (A_1, A_2) of Sets of rational number having the following properties:

$[D_1]$: $A_1, A_2 \neq \phi$

$[D_2]$: $A_1 \cup A_2 = Q$

$[D_3]$: $A_1 < A_2$

$[D_4]$ A_1 does not posses a greatest rational number.

Sum Of two Cuts:
Let $\alpha = (A_1, A_2)$, $\beta = (B_1, B_2)$,
We Construct following two Classes:
(1) The Class C_1 Consisting of all rational numbers C_1 of the form $C_1 = x_1 + y_1$ where x_1 denotes any number of A_1 and y_1 any number of B_1.

(2)

(ii) The class C_1 consisting of all other rational numbers.

Here, we will verify the ordered pair $\gamma = (C_1, C_2)$ is a cut.

(i) Clearly $\gamma = (C_1, C_2) \neq \phi$.

(ii) Let $u \in A_2$ and $v \in V_2$,

u and v are rationals.

then $u > x_1$ and $v > y_1$ so that

$u + v > x_1 + y_1 \; \forall \; x_1 \in A_1$ and $y_1 \in B_1$

$\Rightarrow u + v \in C_2$. So C_1 doesnot contains every rational.

(iii) Let $u \in C_1$ and $t < u$

$\Rightarrow u = p + q$ for some $p \in A_1$ and $q \in B_1$

Let us choose a rational k such that $t = k + q$. then $t < u \Rightarrow k < p$.

then $k \in A_1 \Rightarrow t \in A_1$

(iv) Let $r \in C_1$

then $r = p + q$ for some $p \in A_1$ and $q \in B_1$

Since (A_1, A_2) is a cut then there is rational $u > p$ such that $u \in A_1$.

Thus $u + q \in C_1$. Since $u \in A_1$ and $q \in B_1$

also, $u + q > p + q = r$. So that r is not the largest rational in C_1.

Here C_1 doesnot contain the largest rational. So all the three conditions are satisfied and hence $\gamma = (C_1, C_2)$ is a cut. This cut is called the

∴ Sum of two cuts, i.e,
$(A_1, A_2) + (B_1, B_2)$
$\therefore (C_1, C_2) = (A_1, A_2) + (B_1, B_2)$
i.e, $\gamma = \alpha + \beta$.

Product of Two cuts:
Let us suppose (A_1, A_2) and (B_1, B_2) be two non negative cuts. Let (C_1, C_2) be product of two cuts (A_1, A_2) and (B_1, B_2). Then we form the class C_1 consisting of all rational numbers of the form $c_1 = x_1 y_1$ where $x_1 \in A_1$ and $y_1 \in A_2$ and the class C_2 consisting of all other rational numbers.
We can write $(C_1, C_2) = (A_1, A_2)(B_1, B_2)$
if (A_1, A_2) is negative and (B_1, B_2) is non-negative, then their product is defined as

$$(A_1, A_2)(B_1, B_2) = -\left[\{-(A_1, A_2)\}(B_1, B_2)\right]$$

if (A_1, A_2) and (B_1, B_2) are both negative then we define their product as

$$(A_1, A_2) \cdot (B_1, B_2) = \left[-(A_1, A_2)\right]\left[-(B_1, B_2)\right]$$

The unit Cut:

if I_1 = The set of all rational numbers <1 and I_2 Consists of all other rational numbers. Then $I = (I_1, I_2)$ is called the unit cut.

In other words, the cut corresponding to the rational number 1 is called the unit cut.

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