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B. Sc. Part I (Maths Hons)

Paper-I.

Theory Of Equation.

Theorem :- In an equation with rational coefficients, irrational roots occur in conjugate pairs.

Proof :-

Let, $\alpha+\sqrt{\beta}$ be an irrational root of $f_n(x)=0$. We have to prove that $\alpha-\sqrt{\beta}$ is also a root of the equation $f_n(x)=0$

We divide $f_n(x)$ by $[x-(\alpha+\sqrt{\beta})][x-(\alpha-\sqrt{\beta})]$ i.e $[(x-\alpha)^2-\beta]$. Let the quotient be Q and remainder $Rx+R'$.

Then we have,

$$f_n(x) = [(x-\alpha)^2-\beta]Q + (Rx+R')$$
$$= [x-(\alpha+\sqrt{\beta})][x-(\alpha-\sqrt{\beta})]Q + (Rx+R') \quad (1)$$

Now, since $\alpha+\sqrt{\beta}$ is a root of the eqn $f_n(x)=0$

therefore $f_n(\alpha+\sqrt{\beta})=0$

Putting $x=\alpha+\sqrt{\beta}$ in (1), we get

$f(\alpha+\sqrt{\beta})=0.\ Q+R(\alpha+\sqrt{\beta})+R'$

$\Rightarrow 0 = (R\alpha+R')+R\sqrt{\beta}$, since $f(\alpha+\sqrt{\beta})=0$

$\Rightarrow R\alpha+R'=0,\ R\sqrt{\beta}=0.$

(by equating real and imaginary parts)

$\Rightarrow R\alpha+R'=0$ and $R\sqrt{\beta}=0$

$\therefore R=0,\ R'=0.$

$\therefore f_n(x) = [x-(\alpha+\sqrt{\beta})][x-(\alpha-\sqrt{\beta})]Q$

which shows that

$x-(\alpha-\sqrt{\beta})$ is a factor of $f_n(x)$

$\therefore x=\alpha-\sqrt{\beta}$ is a root of the eqn $f_n(x)=0$

(2)

Q:- A root α of the eqn $3x^3-10x^2+7x+10=0$ Connected with a root α' of the equation $x^3-x^2-17x+65=0$ by the relation — $\alpha\alpha'+\alpha'-\alpha+1=0$. using the fact, solve the two given eqns completely.

Soln:- Since α is a root of the equation
$$3x^3-10x^2+7x+10=0$$
we have,
$$3\alpha^3-10\alpha^2+7\alpha+10=0 \quad \text{— (1)}$$
Again, since α' is a root of the eqn.
$$x^3-x^2-17x+65=0.$$
we have,
$$\alpha'^3-\alpha'^2-17\alpha'+65=0 \quad \text{— (2)}$$
But, $\alpha\alpha'+\alpha'-\alpha+1=0$, i.e $\alpha'(\alpha+1)=\alpha-1$
$$\therefore \alpha'=\frac{\alpha-1}{\alpha+1}$$
Now, putting $\alpha'=\frac{\alpha-1}{\alpha+1}$ in (2), we get
$$\left(\frac{\alpha-1}{\alpha+1}\right)^3-\left(\frac{\alpha-1}{\alpha+1}\right)^2-17\left(\frac{\alpha-1}{\alpha+1}\right)+65=0$$
$$\Rightarrow (\alpha-1)^3-(\alpha-1)^2(\alpha+1)-17(\alpha-1)(\alpha+1)^2+65(\alpha+1)^3=0$$
$$\Rightarrow (\alpha^3-3\alpha^2+3\alpha-1)(\alpha-1)(\alpha^2-1)-17(\alpha^2-1)(\alpha+1)+65(\alpha^3+3\alpha^2+3\alpha+1)=0$$
$$\Rightarrow (\alpha^3-3\alpha^2+3\alpha-1)-(\alpha^3-\alpha^2-\alpha+1)-17(\alpha^3+\alpha^2-\alpha-1)+65(\alpha^3+3\alpha^2+3\alpha+1)=0.$$
$$\Rightarrow \alpha^3(1-1-17+65)+\alpha^2(-3+1-17+195)+\alpha(3+1+17+195)+(-1-1+17+65)=0$$
$$\Rightarrow 48\alpha^3+176\alpha^2+216\alpha+80=0.$$
$$\Rightarrow 6\alpha^3+22\alpha^2+27\alpha+10=0$$
Hence the roots of $3x^3-10x^2+7x+10=0$ are
$$-\frac{2}{3},\ 2+i,\ 2-i.$$
Again, dividing $x^3-17x+65$ by $x+5$, we get $x^2-6x+13$
Solving $x^2-6x+13$, we get $x=\frac{6\pm\sqrt{36-4\cdot1\cdot13}}{2}=\frac{6\pm4i}{2}$
$=3\pm2i$, Hence the roots of $x^3-x^2-17x+65=0$ are $5,\ 3\pm2i$.

(3)

1:- Form of the eqn with rational coefficients which shall have for two of its roots $\sqrt{3}$ and $2+i$.

2/:- We know that in an equation irrational roots and also imaginary roots enter pair.

So it follows that if $\sqrt{3}$ is a root of the given equation, then $-\sqrt{3}$ is also a root. Similarly if $2+i$ be a root of the given eqn. then its conjugate. $2-i$ must be a root.

So the required equation will be

$$(x-\sqrt{3})(x+\sqrt{3})[x-(2+i)][x-(2-i)]=0$$

$$\Rightarrow (x^2-3)[(x-2)^2+1]=0$$

$$\Rightarrow (x^2-3)(x^2-4x+5)=0$$

$$\Rightarrow x^4-4x^3+2x^2+12x-15=0$$

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