

Mathematics.
B.Sc. (Maths Honours)
Part - I
Paper - I

Name of the Topic:
Trigonometry (Hyperbolic
function)

Defⁿ:- Whether z be real or
Complex, the Hyperbolic Sine
and Cosine of z (Written as
 $\sinh z$, $\cosh z$ respectively) are
defined as follows:

$$\sinh z = \frac{e^z - e^{-z}}{2}$$

$$\cosh z = \frac{e^z + e^{-z}}{2}$$

The hyperbolic tangent, cosecant and cotangent are obtained from the hyperbolic sine and cosine just as the circular tangent, cosecant, secant and cotangent are obtained from the circular sine and cosine.

$$\begin{aligned} \text{Thus } \tanh z &= \frac{\sinh z}{\cosh z} \\ &= \frac{e^z - e^{-z}}{e^z + e^{-z}} \end{aligned}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{coth} x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Relation between Circular functions and Hyperbolic functions:

For all values of θ , we have

$$\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta})$$

$$\text{and } \cos \theta = \frac{1}{2} (e^{i\theta} + e^{-i\theta})$$

Hence putting $i\alpha$ for θ , it follows that

$$\sin i\alpha = \frac{1}{2i} (e^{i \cdot i\alpha} - e^{-i \cdot i\alpha})$$

$$= \frac{1}{2i} (e^{-\alpha} - e^{\alpha})$$

$$= -\frac{1}{2i} (e^{\alpha} - e^{-\alpha})$$

$$= \frac{i}{2} (e^{\alpha} - e^{-\alpha}) = i \sinh \alpha.$$

$$\text{and } \cos ix = \frac{1}{2} (e^{ix} + e^{-ix})$$

$$= \frac{1}{2} (e^{-ix} + e^{ix}) = \frac{1}{2} (e^{ix} + e^{-ix})$$

$$= \cosh x.$$

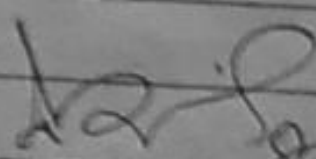
Thus,

$$\left. \begin{aligned} \sin ix &= i \cosh x, \\ \cos ix &= \cosh x, \\ \tan ix &= \frac{\sin ix}{\cos ix} \\ &= i \tanh x \text{ etc.} \end{aligned} \right\} \text{--- (1)}$$

and conversely,

$$\left. \begin{aligned} \sinh x &= \frac{1}{i} \sin ix \\ &= -i \sin ix \\ \cosh x &= \cos ix \\ \text{and } \tanh x &= -i \tan ix \text{ etc.} \end{aligned} \right\} \text{--- (2)}$$

In general corresponding to most trigonometrical formulae involving the circular functions there are formulae involving the hyperbolic functions.


 Dr. B. S. Chandra Prasad
 (Do. Prof.
 Asst. Prof.
 Dept. of Maths.
 D.K. College,
 Madurai
 2021