01.06.2021.

Mathematics.

B.Sc. Maths. Hons

Part - II

Paper - IV

Name of the Topic: Diff. eqn

(Orthogonal trajectory.)

Defn:- A curve which cuts every member of a given family of curves according to a given rule is called a trajectory of the given family.

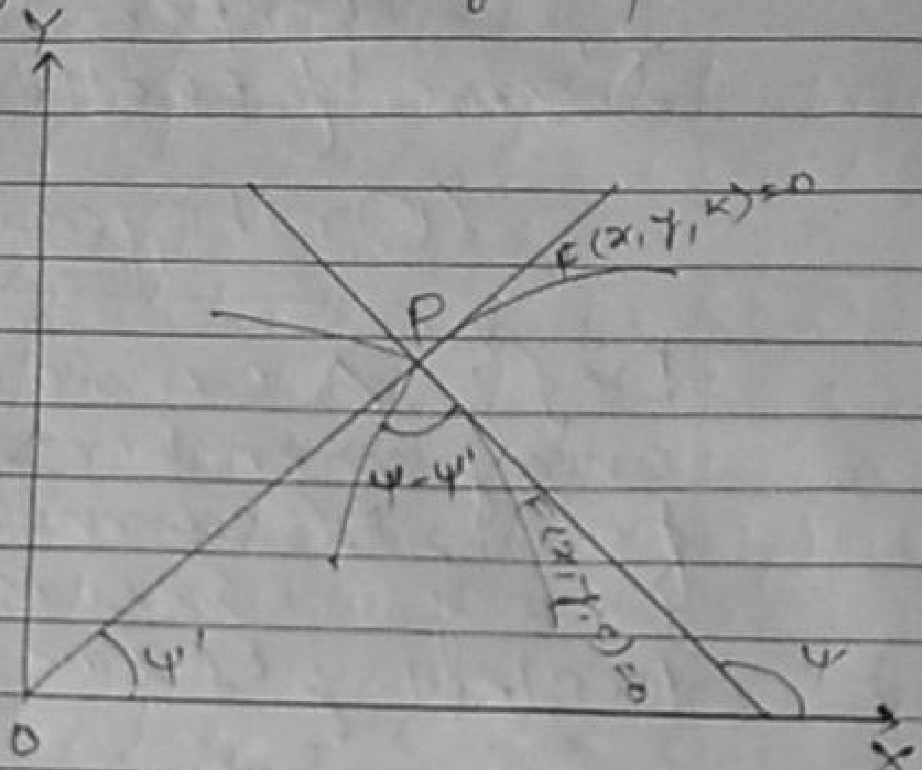
Note:- When each trajectory cuts every member of a given family of curves at a constant angle. if the constant angle is a right angle, then the trajectory is said to be orthogonal.

Rule for finding the orthogonal trajectory.

Orthogonal trajectory

1. Cartesian co-ordinates:-

To find the orthogonal trajectories of the family of curves $f(x, y, c)$; c being a parameter.



The equation of the curves is given to be $f(x, y, c) = 0$ — (1) differentiating (1) w.r.t. to 'x' and and eliminating c, between (1) and the derived result, we shall obtain the differential equation of the given family of curves. The differential equation obtained will evidently involve x, y and $\frac{dy}{dx}$. Let the differential equation be

$$\phi\left(x, y, \frac{dy}{dx}\right) = 0 \text{ — (2)}$$

Let $F(x, y, k) = 0$ be the equation of the curve which cuts the given family $f(x, y, c) = 0$ at a constant angle α.

We know that the angle between the curves is equal to the angle between the tangents at their point of intersection.

Let $m \left(\tan\psi = \frac{dy}{dx}\right)$ and m' denote the slope of the tangents to the curves $f(x, y, c) = 0$ and $F(x, y, k) = 0$ respectively.

Then $\tan\alpha = \dfrac{m - m'}{1 + mm'}$

if $\alpha = 90°$, i.e if the trajectories are orthogonal, then $1 + mm' = 0$

$$\therefore mm' = -1 \Rightarrow m' = \frac{-1}{m}$$

$$= -\frac{1}{\frac{dy}{dx}} = -\frac{dx}{dy}.$$

Thus if $\frac{dy}{dx}$ is the slope of the tangents of the given family of curves, then the slope of the trajectories would be $-\frac{dx}{dy}$

Hence the differential equation of the family of orthogonal trajectories from (2) is $\phi\left(x, y, -\frac{dx}{dy}\right) = 0$