02.06.2021

Mathematics

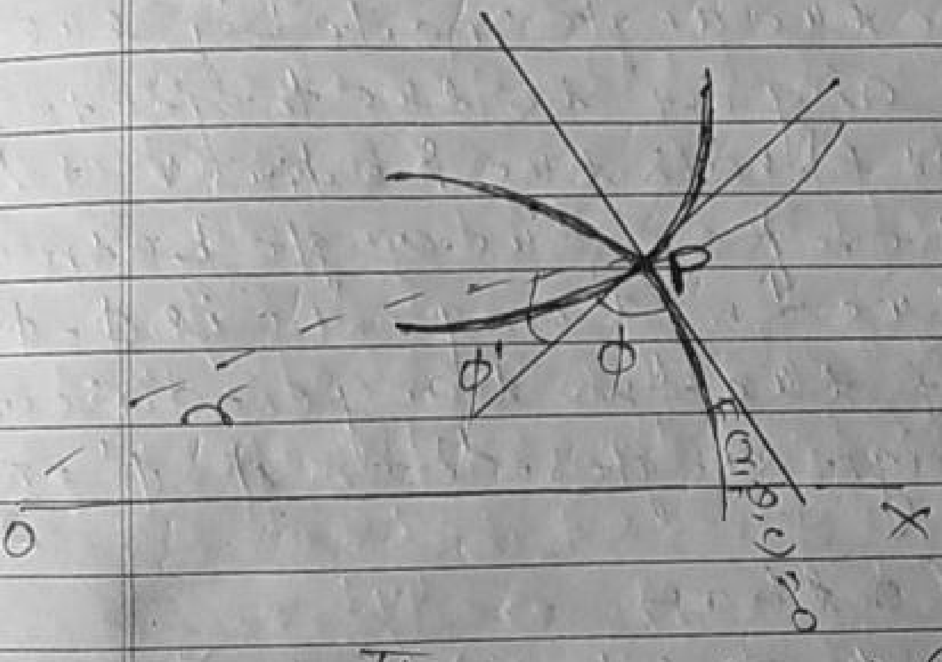
B. Sc. Part-II (Maths HON)

Paper-IV

Name of the Topic:

Polar Co-ordinates (Diff. eqn)

* To find the orthogonal trajectories of the family of curves $f(r, \theta, c) = 0$, c being a parameter.



The equation of the curve is given to be $f(r, \theta, c) = 0$ —(1)
Like before, differentiating (1) w.r.t. θ and eliminating c between (1) and the derived result, we shall obtain the

differential equation of the family. Let the differential equation so obtained be

$$F\left(r, \theta, \frac{dr}{d\theta}\right) = 0. \quad \text{—(2)}$$

Let P be the point of intersection of any member of (1) with the trajectory. Let ϕ be the angle made by the tangent to (1) at P with the (common) radius vector. So that $\tan\phi = r\frac{d\theta}{dr}$. Let ϕ' be the angle made by the tangent to the trajectory at P with the radius vector. Therefore the angle between the two tangents is $\phi - \phi'$. if the two tangents are at right angles. then $\phi - \phi' = \frac{\pi}{2}$.

So that

$$\tan(\phi - \phi') = \tan\frac{\pi}{2}$$

$$\Rightarrow \frac{\tan\phi - \tan\phi'}{1 + \tan\phi \tan\phi'} = \infty$$

Q. 1. Prove that the orthogonal trajectories of the rectangular hyperbola $xy = a^2$ is $x^2 - y^2 = c^2$.

Sol:- Given $xy = a^2$ —(1)

Differentiating (1) w.r.t. x. we get,

$$x\frac{dy}{dx} + y = 0.$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} \quad \text{—(2)}$$

In order to get the orthogonal trajectories of (1). we have to replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in (2).

The differential equation of the orthogonal trajectory of the given curve is

$$-\frac{dx}{dy} = -\frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow x\,dx = y\,dy$$

$$\Rightarrow \int x\,dx = \int y\,dy \Rightarrow \frac{x^2}{2} = \frac{y^2}{2} + k$$

$$\Rightarrow x^2 - y^2 = 2k = c^2 \text{ (by writing } k = \tfrac{c^2}{2}\text{)}$$

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