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Mathematics
B.Sc. Part-I (Maths Hons)
Paper-I.

Name of the Topic: Hyperbolic function (Trigonometry)

(i) Expansion of Sinhx and Coshx.

We have, $\sinh x = -i \sin ix$

$$= -i\left[ix - \frac{(ix)^3}{3!} + \frac{(ix)^5}{5!} - \cdots\right]$$

$$= -i^2\left[x - \frac{i^2x^3}{3!} + \frac{i^4x^5}{5!} - \cdots\right]$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

Similarly,

$$\cosh x = \cos ix$$

$$= \left[1 - \frac{(ix)^2}{2!} + \frac{(ix)^4}{4!} - \cdots\right]$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!}$$

Periods of Hyperbolic funct.

we have,

$$\sinh(\theta + 2n\pi i)$$

$$= -i \sin i(\theta + 2n\pi i)$$

$$= -i \sin(i\theta - 2n\pi)$$

$$= +i \sin(2n\pi - i\theta)$$

$$= -i \sin i\theta = \sinh\theta.$$

Similarly,

$$\cosh(\theta + 2n\pi i)$$

$$= \cosh\theta \text{ and } \tanh(\theta + n\pi i)$$

$$= \tanh\theta.$$

Hence the hyperbolic sine and hyperbolic cosine are periodic, their periods being imaginary and equal to $2n\pi i$. the period of $\tanh\theta$ is half of that of $\sinh\theta$ and $\cosh\theta$.

Ex: Separate the following expression into their real and imaginary parts.

(i) $\sin(x+iy)$

(ii) $\cos(x+iy)$

(iii) $\tan(x+iy)$

Sol?

(i) $\sin(x+iy)$

$= \sin x \cos(iy) + \cos x \cdot \sin(iy)$

$= \sin x \cosh y + i \cos x \sinh y$

(ii) $\cos(x+iy)$

$= \cos x \cos(iy) - \sin x \cdot \sin(iy)$

$= \cos x \cdot \cosh y - i \sin x \cdot \sinh y$

(iii) $\tan(x+iy) = \dfrac{\sin(x+iy)}{\cos(x+iy)}$

$= \dfrac{\sin(x+iy)\cos(x-iy)}{\cos(x+iy)\cos(x-iy)}$

$= \dfrac{\sin 2x + \sin(2iy)}{\cos 2x + \cos(2iy)}$

$= \dfrac{\sin 2x + i \sinh 2y}{\cos 2x + \cosh 2y}$

$= \dfrac{\sin 2x}{\cos 2x + \cosh 2y} + \dfrac{i \sinh 2y}{\cos 2x + \cosh 2y}$

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