07.06.2021

Mathematics.
B.Sc. Part-I (M.H.)
Paper-I.
Topic: Gregory's Series (Higher Trigonometry)

Q. Write down the value of n when θ lies between $\frac{19\pi}{4}$ and $\frac{21\pi}{4}$ for the truth of the general thm.

$$\theta - n\pi = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \cdots\cdots$$

Soln:

We know that the general thm, $\theta - n\pi = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \cdots$ is valid if θ lies between $n\pi - \frac{\pi}{4}$ and $n\pi + \frac{\pi}{4}$.

It is given here that θ lies between $\frac{19\pi}{4}$ and $\frac{21\pi}{4}$

i.e. $5\pi - \frac{\pi}{4}$ and $5\pi + \frac{\pi}{4}$

$$\therefore n = 5.$$

Date
Page

Q. Prove that $\frac{\pi}{2\sqrt{3}} = 1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots$

Solⁿ: The series on the R.H.S. i.e.

$$1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots$$

$$= \sqrt{3}\left[\frac{1}{\sqrt{3}} - \frac{1}{3\cdot(\sqrt{3})^3} + \frac{1}{5}\cdot\frac{1}{(\sqrt{3})^5} - \cdots\right]$$

$$= \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \sqrt{3}\cdot\frac{\pi}{6} = \frac{\pi}{2\sqrt{3}} = \text{L.H.S.}$$

Q. Prove that

$$\frac{\pi}{4} = \left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) + \cdots$$

Solⁿ: R.H.S. $= \left[\frac{2}{3} - \frac{1}{3}\cdot\frac{2}{3^3} + \frac{1}{5}\cdot\frac{2}{3^5} - \cdots\right] + \left[\frac{1}{7} - \frac{1}{3}\cdot\frac{1}{7^3} + \frac{1}{5}\cdot\frac{1}{7^5} - \cdots\right]$

Let $\frac{1}{3} = x$ and $\frac{1}{7} = y$.

Then the R.H.S

$$= 2\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right] + \left[y - \frac{y^3}{3} + \frac{y^5}{5} - \cdots\right]$$

$$= 2\tan^{-1}x + \tan^{-1}y$$

$$= \tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}y$$

$$= \tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}} + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{2}{3} \times \frac{9}{8}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4}\cdot\frac{1}{7}} \quad \left[\text{Here, } \frac{3}{4}\cdot\frac{1}{7} < 1\right]$$

$$= \tan^{-1}\frac{(25/28)}{(25/28)}$$

$$= \tan^{-1}(1)$$

$$= \pi/4.$$ P

Q. Prove that

$$\frac{\pi}{8} = \frac{1}{1\cdot 3} + \frac{1}{5\cdot 7} + \frac{1}{9\cdot 11} + \cdots$$

Sol^n.

$$\text{R.H.S} = \frac{1}{1\cdot 3} + \frac{1}{5\cdot 7} + \frac{1}{9\cdot 11} + \cdots$$

$$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \frac{1}{2}\left(\frac{1}{9} - \frac{1}{11}\right) + \cdots$$

$$= \frac{1}{2}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots\right]$$

$$= \frac{1}{2}\tan^{-1}(1)$$

$$= \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$ Proved

Q. if $x > 0$, Prove that

$$\tan^{-1}x = \frac{\pi}{4} + \left(\frac{x-1}{x+1}\right) - \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \cdots$$

Sol^n: Let $\frac{x-1}{x+1} = y$

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Then R.H.S =

$$\frac{\pi}{4} + y - \frac{1}{3}y^3 + \frac{y^5}{5} - \cdots$$

$$= \frac{\pi}{4} + \tan^{-1} y;$$

provided $-1 \le y \le 1$

i.e, $-1 \le \frac{x-1}{x+1} \le 1$. i.e $x > 0$.

$$= \tan^{-1}(1) + \tan^{-1}\left(\frac{x-1}{x+1}\right)$$

$$= \tan^{-1} \frac{1 + \frac{x-1}{x+1}}{1 - \frac{x-1}{x+1}}$$

Here $\frac{x-1}{x+1} < 1$.

$$= \tan^{-1} \frac{2x}{2}$$

$$= \tan^{-1} x \text{ Pd.}$$

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