07.06.21

MATHEMATICS.

B. Sc. PART - II (M.H.)

Paper - IV

Topic: Singular Solution (Differential Equation)

Ex. Obtain the primitive and singular solution of the equation
$xp^2 - 2yp + 4x = 0.$

Soln:- Given $xp^2 - 2yp + 4x = 0$

i.e, $2yp = xp^2 + 4x$

$$\Rightarrow y = \frac{px}{2} + \frac{2x}{p}.$$

The equation is solvable ~~for y~~ for y.

Differentiating w.r.t. x, we get

$$p = \frac{1}{2}\left\{p \cdot 1 + x\frac{dp}{dx}\right\} + 2\left\{\frac{1}{p}\cdot 1 + x\left(-\frac{1}{p^2}\right)\frac{dp}{dx}\right\}$$

$$\Rightarrow 2p = p + x\frac{dp}{dx} + 4\left\{\frac{1}{p} - \frac{x}{p^2}\frac{dp}{dx}\right\}$$

$$= p + x\frac{dp}{dx} + \frac{4}{p} - \frac{4x}{p^2}\frac{dp}{dx}$$

$$\Rightarrow \frac{dp}{dx}\left\{x - \frac{4x}{p^2}\right\} + \left\{\frac{4}{p} - p\right\} = 0$$

$$\Rightarrow \frac{dp}{dx} \cdot x \left\{ 1 - \frac{4}{p^2} \right\} + \left\{ \frac{4 - p^2}{p} \right\} = 0$$

$$\Rightarrow x \frac{dp}{dx} \left\{ \frac{p^2 - 4}{p^2} \right\} - \frac{(p^2 - 4)}{p} = 0$$

$$\Rightarrow \left(\frac{p^2 - 4}{p} \right) \left\{ \frac{x}{p} \frac{dp}{dx} - 1 \right\} = 0.$$

$$\therefore \frac{x}{p} \frac{dp}{dx} = 1$$

$$\Rightarrow \frac{dp}{p} = \frac{dx}{x}$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{dx}{x}$$

$$\Rightarrow \log p = \log x + \log c$$

$$\therefore p = cx$$

Eliminating p, we get the complete primitive as

$$x \cdot c^2 x^2 - 2y \cdot cx + 4x = 0$$

$$\Rightarrow c^2 x^3 - 2cyx + 4x = 0$$

$$\Rightarrow c^2 x^2 - 2cy + 4 = 0$$

$$\therefore \text{C - disc. is } 4y^2 - 4x^2 \cdot 4 = 0$$

i.e. $y^2 - 4x^2 = 0$.

Also p disc. is $y^2 - 4x^2 = 0$.
Clearly $y^2 - 4x^2 = 0$. i.e
$y = \pm 2x$ which occurs once in both p and c - discriminants is the singular solution.

Reduce the equation $p^3 - 4xyp + 8y^2 = 0$ to Clairaut's form by the substitution $y = v^2$. Hence solve the equation to find singular solution.

Sol^n:- Put $y = v^2$. So that

$$p = \frac{dy}{dx} = 2v\frac{dv}{dx} = 2vp$$

where $p = \frac{dv}{dx}$

$\therefore$ From the given equation

$$8v^3p^3 = 4xv^2\, 2vp - 8v^4$$

$$\Rightarrow v^3p^3 = xv^3p - v^4$$

$$\Rightarrow p^3 = xp - v$$

$$\therefore v = px - p^3$$

which is the Clairaut's form and therefore its solution is

$$v = kx - k^3$$

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$\Rightarrow \sqrt{y} = k(x-k^2)$

$\Rightarrow y = k^2(x-k^2)^2$

$\Rightarrow y = c(x-c)^2$

where $k^2 = c$

Here, it is not a quadratic function in c and hence its c-discriminant will be obtained by using $f(c) = 0$ and $f'(c) = 0$.

Differentiating w.r.t. c, treating x, y as constants, we get

$0 = c \cdot 2(c-x) + (c-x)^2 \cdot 1$

$\Rightarrow (c-x)(3c-x) = 0$

$\therefore c = x$ or $x/3$.

when $c = x$, then $y = 0$ and if $c = \frac{x}{3}$ then $y = \frac{4x^3}{27}$

Hence c-disc. is $y(4x^3 - 27y) = 0$ — (i) $[ENC^2S]$

Again, for the p-discriminant, we have

$p^3 - 4xyp + 8y^2 = 0$

Differentiating w.r.t. p,

treating x, y as constants, we get

$$3p^2 - 4xy = 0$$

$$\Rightarrow p^2 = \frac{4}{3}xy$$

Eliminating p, we get.

$$p(p^2 - 4xy) + 8y^2 = 0$$

$$\Rightarrow p\left(\frac{4xy}{3} - 4xy\right) + 8y^2 = 0.$$

$$\Rightarrow p\left(-\frac{8xy}{3}\right) = -8y^2$$

$$\Rightarrow p^2x^2y^2 = 9y^4$$

$$\Rightarrow \frac{4xy}{3}\cdot x^2y^2 - 9y^4 = 0$$

$$\Rightarrow 4x^3y^3 - 27y^4 = 0$$

$$\Rightarrow y^3(4x^3 - 27y) = 0$$

$$\Rightarrow y^2y(4x^3 - 27y) = 0 \quad (2) \; [ET^2C]$$

Hence from (1) and (2), $y = 0$ and $4x^3 - 27y = 0$ are the singular solutions and $y = 0$ which occurs twice in p-disc, is taclocus

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