08.06.2021

Mathematics Hons
B. Sc. Part-II
Paper-IV

Topic: Linear equations with Constant coefficients (Diff. eqn)

Linear equations are those equations in which dependent variable and its derivatives appear only in the first degree and are not multiplied together.

The general differential equation of the nth order is

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = X.$$

Where X and the coefficients $P_1, P_2, P_3 \ldots P_n$ are functions of x only.

Here the equation is linear since the dependent variable y and its differential coefficients occur in the first degree. If $P_1, P_2, P_3 \ldots P_n$ are constants and X be the function of x alone, then the above equation is said to be the linear differential equation with constants coefficients.

Linear Equations with Constant Coefficients:

Let the general linear equation with constant coefficients be of the form

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_n y = X \quad \text{--- (1)}$$

where $P_1, P_2, \dots P_n$ are constants and X is a function of x for a constant.

Theorem I:-

If y_1, y_2 are any two solution of

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_n y = 0 \quad \text{--- (2)}$$

$$= C_1\left(\frac{d^n y_1}{dx^n} + P_1\frac{d^{n-1}y_1}{dx^{n-1}} + P_2\frac{d^{n-2}y_1}{dx^{n-2}} + \cdots\right)$$

$$+ C_2\left(\frac{d^n y_2}{dx^n} + P_1\frac{d^{n-1}y_2}{dx^{n-1}} + P_2\frac{d^{n-2}y_2}{dx^{n-2}} + \cdots\right)$$

$$= C_1 \times 0 + C_2 \times 0$$

(because of ③ ~~and~~ ④)

$$= 0.$$

This proves the theorem.

Symbolic Represen-tation of

$$\frac{d^n y}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + P_n y = 0$$

Let, D Stands for $\frac{d}{dx}$

D^2 for $\frac{d^2}{dx^2}$ ...

and D^n for $\frac{d^n}{dx^n}$.

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