

11.06.2021

Mathematics. Hons.

B.Sc. Part - II

Paper - IV

Topic: Linear equations with  
constant coefficients  
(diff. eqn.)

CASE III! - Roots conjugate  
complex and not repeated.

Let  $m_1 = \alpha + i\beta$  and  
 $m_2 = \alpha - i\beta$  be the roots of  
the auxiliary equation. Then  
the general solution of (4) is

$$y = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$$

$$y = C_1 e^{\alpha x} e^{i\beta x} + C_2 e^{\alpha x} e^{-i\beta x}$$

$$= e^{\alpha x} \{ C_1 e^{i\beta x} + C_2 e^{-i\beta x} \}$$

$$= e^{\alpha x} \{ C_1 (\cos \beta x + i \sin \beta x) \}$$

$$+ C_2 (\cos \beta x - i \sin \beta x)$$

$$= e^{\alpha x} \{ (C_1 + C_2) \cos \beta x +$$

$$i(C_1 - C_2) \sin \beta x \}$$

$$= e^{\alpha x} (A \cos \beta x + B \sin \beta x)$$

Where  $A = C_1 + C_2$  and  $B = i(C_1 - C_2)$

are arbitrary constants.

Note! Let it be noted that  $\alpha$  is the real part and  $\beta$  is the imaginary part of the complex root  $\alpha \pm i\beta$ .

CASE IV:- Roots conjugate complex and repeated.

When the conjugate complex roots  $\alpha \pm i\beta$  are each repeated  $r$  times, the corresponding part of the value of the complementary function is

$$e^{\alpha x} \{ A_1 + A_2 x + \dots + A_r x^{r-1} \} \\ \cos \beta x + \{ B_1 + B_2 x + \dots + B_r x^{r-1} \} \\ \sin \beta x \}$$

It is an extension of the Case No. III.

Q3) Solve  $\frac{d^2 y}{dx^2} + 4y = 0$

Sol<sup>n</sup>:- The auxiliary eq<sup>n</sup> is  $m^2 + 4 = 0$

$$\Rightarrow m^2 = -4$$

$$\Rightarrow m = \pm 2i$$

(Here real part  $= 0$  and  
imaginary part  $= 2$ )

∴ The general solution is

$$y = e^{0 \cdot x} (C_1 \cos 2x + C_2 \sin 2x)$$

$$\text{i.e., } y = C_1 \cos 2x + C_2 \sin 2x$$

Ex Solve  $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

Sol<sup>n</sup>:- The auxiliary eq<sup>n</sup> is  
 $m^2 + m + 1 = 0$ .

Solving the equation, we get

$$m = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

Here real part  $= -\frac{1}{2}$  and

imaginary part  $= \frac{\sqrt{3}}{2}$

∴ The general solution is

$$y = e^{-\frac{1}{2}x} \left( C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x \right)$$

Ex Solve  $\frac{d^3y}{dx^3} - 8y = 0$ .

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Sol<sup>n</sup>: The auxiliary eq<sup>n</sup> is

$$m^3 - 8 = 0$$

$$\Rightarrow (m-2)(m^2 + 2m + 4) = 0$$

$$\therefore m = 2 \text{ or } m^2 + 2m + 4 = 0$$

Solving  $m^2 + 2m + 4 = 0$ , we get

$$m = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 4}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}i}{2}$$

$$= -1 \pm \sqrt{3}i$$

Hence  $m = 2, -1 \pm \sqrt{3}i$

(real part = 1 and imaginary part =  $\sqrt{3}$ )

$\therefore$  The general solution is

$$y = C_1 e^{2x} + e^{-x} (C_2 \cos \sqrt{3}x + C_3 \sin \sqrt{3}x).$$

~~Q. 2~~

Solve  $\frac{d^4 y}{dx^4} - 4y = 0$

Sol<sup>n</sup>

The auxiliary equation is

$$m^4 - a^4 = 0.$$

$$\Rightarrow (m^2 - a^2)(m^2 + a^2) = 0.$$

$$\text{from } m^2 - a^2 = 0$$

$$m^2 = a^2$$

$$\text{and } m^2 + a^2 = 0$$

$$\therefore m^2 = -a^2$$

$$\Rightarrow m = \pm a, \pm ia.$$

Therefore the general solution is

$$y = C_1 e^{ax} + C_2 e^{-ax} + (C_3 \cos ax + C_4 \sin ax).$$

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