12.06.2021

Mathematics Hons.

B. Sc. Part-II

Paper - IV

Topic: Particular Integral (Diff. eqns)

The methods of obtaining particular Integrals of non-homogeneous partial differential equations are very similar to those used in solving linear equations with constant coefficients. Here we are considering few cases.

CASE I: When the R.H.S. of differential equation is of the form e^{ax+by}.

Then $\frac{1}{f(D,D')} e^{ax+by} = \frac{1}{f(a,b)} e^{ax+by}$ provided $f(a,b) \neq 0$

i.e, put $D = a$ and $D' = b$.

CASE II: When the R.H.S of the differential equation is of the form $\sin(ax+by)$ or $\cos(ax+by)$.

Then $\frac{1}{f(D,D')} \sin(ax+by)$

Or $\cos(ax+by)$ is obtained by putting $D^2 = -a^2$, $DD' = -ab$, $D'^2 = -b^2$ provided the denominator is not zero.

CASE III : When the R.H.S of the differential equation is of the form $x^m y^n$ where m and n are positive integers then

$$\frac{1}{f(D,D')} x^m y^n = \left[f(D,D')\right]^{-1} x^m y^n$$

CASE IV When the R.H.S of the differential equation is of the form
$$e^{ax+by} V.$$
Then
$$\frac{1}{f(D,D')} e^{ax+by} V$$
$$= e^{ax+by} \frac{1}{f(D+a, D'+b)} V.$$

Some Examples:

(1) Solve: $(D^2 - D'^2 + D - D')z = 0$

Soln: When factorised

$$D^2 - D'^2 + D - D'$$
$$= (D^2 - D'^2) + (D - D')$$
$$= (D - D')(D + D')(D - D')$$
$$= (D - D')(D + D' + 1)$$

Hence the given equation is

$(D - D')(D + D' + 1)z = 0.$

$\therefore z = f_1(y + x) + e^{-x} f_2(y - x)$

(2) Solve: $DD'(D - 2D' - 3)z = 0$

Soln:- The given equation can be written as

$(D - 0 \cdot D' - 0)(0 + D' - 0)(D - 2D' - 3)z = 0$

$\therefore z = e^{0 \cdot x} f_1(y + 0 \cdot x) + e^{0 \cdot x} f_2(0 - x) + e^{3x} f_3(y + 2x)$

$= f_1(y) + \phi(x) + e^{3x} f_3(y + 2x)$

(3) Solve: $r + 2s + t + 2p + 2q + z = 0$

Soln: The above equation can be written as

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} + z = 0.$$

$\Rightarrow (D^2 + 2DD' + D'^2 + 2D + 2D' + 1)z = 0.$

$\Rightarrow (D + D' + 1)^2 z = 0.$

Hence $z = e^{-x} f_1(y-x) + x e^{-x} f_2(y-x)$

(4) Solve: $(2D^4 - 3D^2D' + D'^2)z = 0$

Soln:

The given equation can be written as

$(2D^2 - D')(D^2 - D')z = 0.$

However, non of factors can be further resolved into factors linear in D and D'.

Hence let $z = Ae^{hx+ky}$ be the C.F. corresponding to

$(D^2 - D')z = 0$

$\Rightarrow (D^2 - D')z = Ah^2 e^{hx+ky} - Ak e^{hx+ky}$

$= A(h^2 - k) e^{hx+ky}$ —— (1)

∴ The given equation (1) will

be satisfied by the substitution if $h^2-k=0$.

$\therefore$ The C.F corresponding to $(D^2-D')z=0$ is

$\sum Ae^{hx+h^2y}$.

Similarly C.F corresponding to $(2D^2-D')z=0$ is

$\sum Be^{h'x+k'y}$

where $2h'^2-k'=0$ i.e

$\sum Be^{h'x+2h'^2y}$.

$\therefore$ The most general solution of the given equation is

$$z=\sum Ae^{hx+h^2y}+\sum Be^{h'x+2h'^2y}$$

(5) Solve $(D-2D'-1)(D-2D'^2-1)z=0$.

Sol^n: The C.F corresponding to the first factor is $z=e^x f_1(y+2x)$

The C.F corresponding to the 2nd factor is $\sum Ae^{hx+ky}$.

where $h-2k^2-1=0$ i.e. $h=2k^2+1$

Hence $z=e^x f_1(y+2x)+\sum Ae^{hx+ky}$ where $h=2k^2+1$.

Sd/-

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