14.06.2021.

Mathematics Hons.
B.Sc. Part II
Paper - IV
Topic: Integral (Diff. eqn)

(1) Solve: $r - s + p = 1$.

Soln:-

The given equation can be written as

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 1.$$

i.e $(D^2 - DD' + D)Z = 1$.

i.e $D(D - D' + 1)Z = 1$.

$C.F = e^{0.x} f_1(y) + e^{-x} f_2(y+x)$

$= f_1(y) + e^{-x} f_2(y+x)$

Now, $P.I = \frac{1}{D(D-D'+1)}(1)$

$= \frac{1}{D}(1 + D - D')^{-1}(1)$

$= \frac{1}{D}(1 - D + D' + \cdots)(1)$

$= \frac{1}{D}(1) = x$.

∴ The solution is

$Z = f_1(y) + e^{-x} f_2(y+x) + x$.

(2) Solve: $(D^3-3DD'+D+1)z = e^{2x+3y}$.

Solⁿ: Here we see that $D^3-3DD'+D+1$ cannot be resolved into factors linear in D and D'.

$\therefore$ C.F $= \sum Ae^{hx+ky}$ where

$h^3-3hk+h+1=0$

Also P.I $= \dfrac{1}{D^3-3DD'+D+1} e^{2x+3y}$

$= \dfrac{e^{2x+3y}}{2^3-3.2.3+2+1} = \dfrac{e^{2x+3y}}{-7}$

$\therefore$ The required solution is

$z = -\frac{1}{7} e^{2x+3y} + \sum Ae^{hx+ky}$.

Where $h^3-3hk+h+1=0$.

(3) Solve: $S+p-q = z+xy$

Solⁿ: The given equation can be written as

$\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y} - z = xy$

i.e, $(DD'+D-D'-1)z = xy$

$\Rightarrow (D-1)(D'+1)z = xy$

$\therefore C.F = e^{x} f_1(y) + e^{-y} f_2(x)$

Now, $P.I = \frac{1}{(D-1)(D'+1)} xy$

$= \frac{1}{-(1-D)(1+D')} xy$

$= -(1-D)^{-1}(1+D')^{-1} xy$

$= -\left[(1+D+D^2+\ldots)(1-D'+D'^2-\ldots)\right] xy$

$= -\left[1+D-D'-DD'-\ldots\right] xy$

$= -\left[xy+y-x-1\right]$

Hence the solution is

$Z = e^{x} f_1(y) + e^{-y} f_2(x) - xy - y + x + 1$

(4) Solve: $(D^2 - DD' - 2D)z = \sin(3x+4y)$

Solⁿ:- Here, $D^2 - DD' - 2D$ cannot be resolved into factors linear in D and D'.

$\therefore C.F = \Sigma A e^{hx+ky}$ where

$h^2 - hk - 2h = 0 \Rightarrow h - k - 2 = 0$

Now, $P.I = \frac{1}{D^2 - DD' - 2D} \sin(3x+4y)$

$$= \frac{1}{-3^2 - (-3 \cdot 4) - 2D} \sin(3x+4y)$$

$$= \frac{1}{-9+12-2D} \sin(3x+4y)$$

$$= \frac{1}{3-2D} \sin(3x+4y)$$

$$= \frac{3+2D}{9-4D^2} \sin(3x+4y)$$

$$= \frac{3+2D}{9-4(-9)} \sin(3x+4y)$$

$$= \frac{1}{45}(3+2D) \sin(3x+4y)$$

$$= \frac{1}{45}\left[3\sin(3x+4y) + 2 \cdot 3\cos(3x+4y)\right]$$

$$= \frac{1}{15}\left[\sin(3x+4y) + 2\cos(3x+4y)\right]$$

Hence the required solution is

$$z = \frac{1}{15}\left[\sin(3x+4y) + 2\cos(3x+4y)\right] + \sum A e^{hx+ky}$$

where $h-k=2$.

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(5) Solve: $(D-3D'-2)^2 z = 2e^{2x}\tan(y+3x)$

Sol^n: Here $C.F = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x)$

Now, $P.I = \dfrac{1}{(D-3D'-2)^2} \cdot 2e^{2x}\tan(y+3x)$

$= \dfrac{1}{(D-3D'-2)^2} \cdot 2e^{2x+0.y}.\tan(y+3x)$

$= 2e^{2x+0.y} \dfrac{1}{[(D+2)-3(D'+0)-2]^2} \cdot \tan(y+3x)$

$= 2e^{2x} \cdot \dfrac{1}{(D-3D')^2} \cdot \tan(y+3x)$

$= 2e^{2x} \dfrac{x^2}{1^2 \cdot 2!} \tan(y+3x)$

~~where~~ Here $F(a,b) = 0$

$= x^2 e^{2x} \tan(y+3x)$

Hence the solution is $z = C.F + P.I$

i.e, $z = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x) + x^2 e^{2x} \tan(y+3x)$

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