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Date
Page

Mathematics Hons.
B. Sc. Part-I
Paper-I

Topic: C + iS Method.
(Summation of Trigonometrical Series)

Use Of Exponential Series

We shall use the following expansions,

(i) $e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$ to ∞

(ii) $e^{-x} = 1 - x + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \cdots$ to ∞

(iii) $\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots$ to ∞

(iv) $\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots$ to ∞

(v) $\sinh x = x + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \cdots$ to ∞

(vi) $\cosh x = 1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \cdots$ to ∞

Ex. Sum the Series:

$$\sin\theta - \frac{\sin 2\theta}{\lfloor 2} + \frac{\sin 3\theta}{\lfloor 3} \ldots \text{ad inf.}$$

Sol^n: Let $C = \cos\theta - \frac{\cos 2\theta}{\lfloor 2} + \frac{\cos 3\theta}{\lfloor 3} \ldots$ $\ldots$ to ∞.

and $S = \sin\theta - \frac{\sin 2\theta}{\lfloor 2} + \frac{\sin 3\theta}{\lfloor 3} \ldots$ $\ldots$ to ∞.

Then $C + iS$

$$= (\cos\theta + i\sin\theta) - \frac{1}{\lfloor 2}(\cos 2\theta + i\sin 2\theta) + \frac{1}{\lfloor 3}(\cos 3\theta + i\sin 3\theta) \ldots \text{to inf.}$$

$$= e^{i\theta} - \frac{1}{\lfloor 2} e^{i2\theta} + \frac{1}{\lfloor 3} e^{i3\theta} - \ldots \text{to inf.}$$

$$= x - \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots \text{to inf.}$$

On putting $e^{i\theta} = x$.

Now, $e^{-x} = 1 - x + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \ldots$ to inf.

So that $x - \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} - \ldots$ to inf.

$$= 1 - e^{-x}.$$

$\therefore C + iS = 1 - e^{-x}$

$= 1 - e^{-e^{i\theta}}$

$= 1 - e^{-(\cos\theta + i\sin\theta)}$

$= 1 - e^{-\cos\theta} \cdot e^{-i\sin\theta}$

$= 1 - e^{-\cos\theta}[\cos(\sin\theta) - i\sin(\sin\theta)]$

$= 1 - e^{-\cos\theta}\cos(\sin\theta) + ie^{-\cos\theta} \cdot \sin(\sin\theta)$

Hence $S = e^{-\cos\theta}\sin(\sin\theta)$.

~~Ex~~ Sum the Series

$$\cos\theta + \frac{\sin\theta}{\lfloor 1}\cos 2\theta + \frac{\sin^2\theta}{\lfloor 2}\cos 3\theta + \frac{\sin^3\theta}{\lfloor 3}\cos 4\theta + \cdots \text{ ad inf.}$$

Sol^n: Let $C = \cos\theta + \frac{\sin\theta}{\lfloor 1}\cos 2\theta + \frac{\sin^2\theta}{\lfloor 2}\cos 3\theta + \frac{\sin^3\theta}{\lfloor 3}\cos 4\theta + \cdots$ to inf.

and $S = \sin\theta + \frac{\sin\theta}{\lfloor 1}\sin 2\theta + \frac{\sin^2\theta}{\lfloor 2}\sin 3\theta + \frac{\sin^3\theta}{\lfloor 3}\sin 4\theta + \cdots$ to inf.

Then $C + iS = (\cos\theta + i\sin\theta) + \frac{\sin\theta}{\lfloor 1}(\cos 2\theta + i\sin 2\theta)$

$+ \frac{\sin^2\theta}{\lfloor 2}(\cos 3\theta + i\sin 3\theta)$

$+ \cdots$ to inf.

$$= e^{i\theta} + \frac{\sin\theta}{\lfloor 1} e^{i2\theta} + \frac{\sin^2\theta}{\lfloor 2} e^{i3\theta} + \cdots \text{ to inf.}$$

$$= e^{i\theta}\left[1 + \frac{\sin\theta}{\lfloor 1} e^{i\theta} + \frac{\sin^2\theta}{\lfloor 2} e^{i2\theta} + \cdots \text{ to inf.}\right]$$

$$= e^{i\theta}\left[1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \cdots \text{ to inf.}\right]$$

On Putting $\sin\theta\, e^{i\theta} = x$

$$= e^{i\theta}.e^{x} = e^{i\theta}\, e^{\sin\theta.\, e^{i\theta}}$$

$$= e^{i\theta}.\, e^{\sin\theta(\cos\theta + i\sin\theta)}$$

$$= e^{i\theta}.\, e^{\sin\theta\cos\theta}.\, e^{i\sin^2\theta}$$

$$= e^{\sin\theta\cos\theta}.\, e^{i(\theta + \sin^2\theta)}$$

$$= e^{\sin\theta.\cos\theta}\left[\cos(\theta + \sin^2\theta) + i\sin(\theta + \sin^2\theta)\right]$$

Hence $C = e^{\sin\theta\cos\theta}.\cos(\theta + \sin^2\theta)$

and $S = e^{\sin\theta.\cos\theta}\sin(\theta + \sin^2\theta)$

Dr. Birer Prasad
Asst. Prof.
Dept of Maths.
D.K. College,
Dumraon