

17.06.2021.

MATHS.  
(Sub. of Gen)  
Part - I

Topic: Linear Programming  
(Convex Set)

Def<sup>n</sup>.

1. Euclidean Space or  
 $n$ -dimensional vector  
Space  $R^n$ :

The Set in which every element or point is represented by means of  $n$ -coordinates is the set of all ordered  $n$ -tuples of real numbers  $(x_1, x_2, x_3, \dots, x_n)$  is called the  $n$ -dimensional space  $R^n$  of real numbers. This is a vector space under the two compositions of addition and scalar multiplication.

2. Addition and multiplication

$$\text{if } x = (x_1, x_2, x_3, \dots, x_n)$$

$$y = (y_1, y_2, y_3, \dots, y_n)$$

$$\text{Then } x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3, \dots, x_n + y_n)$$

and  $\lambda x = (\lambda x_1, \lambda x_2, \lambda x_3, \dots, \lambda x_n)$   
where  $\lambda$  is a scalar.

### B. Inner product of vectors:

For every pair of vectors  $u$  and  $v$  in  $\mathbb{R}^n$  defined by  
 $u = (u_1, u_2, u_3, \dots, u_n)$   
and  $v = (v_1, v_2, v_3, \dots, v_n)$   
the number  $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3 + \dots + u_n v_n = \sum u_i v_i$   
is called the inner product of  $u$  and  $v$ .

A Line Segment: We know the equation of a line passing through two points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  is

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2} = \lambda \quad (\text{say})$$

then,  $\frac{x - x_1}{x_1 - x_2} = \lambda$

Or,  $x - x_1 = \lambda (x_1 - x_2)$

Or,  $x = \lambda x_1 + (1 - \lambda) x_2$

Similarly, from  $\frac{y - y_1}{y_1 - y_2} = \lambda$

we get,

$$y = \lambda y_1 + (1-\lambda)y_2$$

Thus, we get

$$x = \lambda x_1 + (1-\lambda)x_2$$

$$\text{and } y = \lambda y_1 + (1-\lambda)y_2$$

Thus if  $x \in \mathbb{R}^n$ ,  $y \in \mathbb{R}^n$  and  $x \neq y$ , then the set of points of  $\mathbb{R}^n$  defined as

$Z = \{z : z = \lambda x + (1-\lambda)y, 0 \leq \lambda \leq 1\}$  is called the line segment joining the points  $x, y$  in  $\mathbb{R}^n$ .

Hyper Plane: if the set of points  $x = (x_1, x_2, x_3, \dots, x_n) \in \mathbb{R}^n$  satisfy the relation

$$c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_n x_n = a,$$

where  $c = (c_1, c_2, c_3, \dots, c_n) \neq 0$  and  $a \in \mathbb{R}$  are pre assigned. In particular, if  $x = (x_1, x_2, x_3) \in \mathbb{R}^3$  then the set of point  $\mathbb{R}^3$  which satisfy the relation  $c_1 x_1 + c_2 x_2 + c_3 x_3 = a$ , is called a hyperplane in three dimension where  $c_1, c_2, c_3$  are all non-zero at the same time and  $a$  is a real constant.

Dr. J. S. G. D. S. G.  
Asst. Prof. Math.  
Dept. of Math.  
D.K. College,  
Dumraon