17.06.2021.

Mathematics Hons.

B. Sc. Part-I

Paper-I

Topic: Summation of Trigonometrical Series.

Ex. (1) Sum the Series

$$1-\cos\alpha\cos\beta+\frac{\cos^2\alpha}{\lfloor 2}\cos 2\beta-\frac{\cos^3\alpha}{\lfloor 3}\cos 3\beta+\cdots \text{ to } \infty.$$

Soln:- Let $C = 1-\cos\alpha\cos\beta+\frac{\cos^2\alpha}{\lfloor 2}\cos 2\beta-\frac{\cos^3\alpha}{\lfloor 3}\cos 3\beta+\cdots$

and $S = -\cos\alpha\sin\beta+\frac{\cos^2\alpha}{\lfloor 2}\sin 2\beta-\frac{\cos^3\alpha}{\lfloor 3}\sin 3\beta+\cdots$

Then $C+iS$

$$= 1-\cos\alpha\cdot e^{i\beta}+\frac{\cos^2\alpha}{\lfloor 2}e^{i2\beta}-\frac{\cos^3\alpha}{\lfloor 3}e^{i3\beta}+\cdots$$

$$= 1-x+\frac{x^2}{\lfloor 2}-\frac{x^3}{\lfloor 3}+\cdots$$

On Putting $\cos\alpha\cdot e^{i\beta}=x$.

$$= e^{-x} = e^{-\cos\alpha\cdot e^{i\beta}}$$

$= e^{-\cos\alpha(\cos\beta + i\sin\beta)}$

$= e^{-\cos\alpha\cos\beta - i\cos\alpha\sin\beta}$

$= e^{-\cos\alpha\cos\beta} \cdot e^{-i\cos\alpha\sin\beta}$

$= e^{-\cos\alpha\cos\beta}\{\cos(\cos\alpha\sin\beta) - i\sin(\cos\alpha\sin\beta)\}$

Now, equating the real part, we get

$$C = e^{-\cos\alpha\cos\beta}\cos(\cos\alpha\sin\beta)$$

~~6~~ Sum the infinite series

$$x\sin(\alpha+\beta) + \frac{x^2}{\lfloor 2}\sin(\alpha+2\beta) + \frac{x^3}{\lfloor 3}\sin(\alpha+3\beta) + \cdots$$

and $$x\cos(\alpha+\beta) + \frac{x^2}{\lfloor 2}\cos(\alpha+2\beta) + \frac{x^3}{\lfloor 3}\cos(\alpha+3\beta) + \cdots$$

Solⁿ:-

We denote the first series by S and the second series by C and multiplying the first series by i, we add it to the second. Thus

$$C + iS = x\{\cos(\alpha+\beta) + i\sin(\alpha+\beta)\} + \frac{x^2}{\lfloor 2}\{\cos(\alpha+2\beta) + i\sin(\alpha+2\beta)\} + \frac{x^3}{\lfloor 3}\{\cos(\alpha+3\beta) + i\sin(\alpha+3\beta)\} + \cdots$$

$$\Rightarrow C+iS = x e^{i(\alpha+\beta)} + \frac{x^2}{\lfloor 2} e^{i(\alpha+2\beta)} + \frac{x^3}{\lfloor 3} e^{i(\alpha+3\beta)} + \cdots$$

$$= e^{i\alpha}\left\{ x e^{i\beta} + \frac{(x e^{i\beta})^2}{\lfloor 2} + \frac{(x e^{i\beta})^3}{\lfloor 3} + \cdots \right\}$$

$$= e^{i\alpha}\left\{ y + \frac{y^2}{\lfloor 2} + \frac{y^3}{\lfloor 3} + \cdots \right\}$$

On putting $x e^{i\beta} = y$

$$= e^{i\alpha}(e^{y}-1) = e^{i\alpha}(e^{x e^{i\beta}}-1)$$

$$= e^{i\alpha}\{ e^{x(\cos\beta + i\sin\beta)} - 1 \}$$

$$= e^{i\alpha}\{ e^{x\cos\beta} \cdot e^{ix\sin\beta} - 1 \}$$

$$= e^{x\cos\beta} \cdot e^{i(\alpha + x\sin\beta)} - e^{i\alpha}$$

$$= e^{x\cos\beta}\{\cos(\alpha + x\sin\beta) + i\sin(\alpha + x\sin\beta)\} - \{\cos\alpha + i\sin\alpha\}$$

Now

Equating the real and imaginary parts separately, we get

$$C = e^{x\cos\beta} \cdot \cos(\alpha + x\sin\beta) - \cos\alpha$$

$$S = e^{x\cos\beta} \cdot \sin(\alpha + x\sin\beta) - \sin\alpha$$

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