

18.06.2021

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Mathematics Hon. & Sub.

B.Sc. Part-I

Paper-1.

Topic: Linear Programming
(Convex Set)

Convex Set: A Subset A of the Euclidean space R^n is called a convex set if for every pair $x_1, x_2 \in A$, the line segment

$$V = \lambda x_1 + (1-\lambda)x_2$$
, where $0 \leq \lambda \leq 1$ lies in A . i.e., $V \in A$.

This can be written as

$x_1, x_2 \in A \Rightarrow \lambda_1 x_1 + \lambda_2 x_2 \in A$.

Where $\lambda_1, \lambda_2 \geq 0$ and $\lambda_1 + \lambda_2 = 1$.

Thus A is convex set if convex combination of every pair of points of A is also a point of A .

Example:

(1) The set of points in the interior of a circle.

(2) The set of points of the two dimensional plane R^2

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Extreme points of a

Convex Set: x is the extreme point of the convex set A , if it is impossible to find $y, z, (\neq x) \in A$. Such that
 $x = \lambda y + (1-\lambda)z; 0 < \lambda < 1$.

The vertices of a triangle are the only three extreme points of the convex set of points on the triangle and its interior and every point on the boundary of a circle is an extreme-point of the convex set.

Convex Combination!

Let $x_1, x_2, x_3, \dots, x_m$ be m points in \mathbb{R}^n and $x \in \mathbb{R}^n$ such that

$$x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \dots + \lambda_m x_m \\ = \sum_{i=1}^m \lambda_i x_i$$

where the λ_i 's are scalars, it is called a convex combination of the given vector if
 $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m \geq 0$

and $\sum_{i=1}^m \lambda_i = 1$.

Thus a line segment
[$x = \lambda_1 x_1 + \lambda_2 x_2, \lambda_1, \lambda_2 \geq 0$
and $\lambda_1 + \lambda_2 = 1$]

Joining two points x_1 and x_2
is the set of all convex combinations of x_1 and x_2 .

Convex Cone! A non-empty set $S \subset \mathbb{R}^n$ is called a cone if for each x in S and $\lambda \geq 0$, the vector λx is also in S .

A cone is called a convex cone if it is a convex set. Thus S is a convex cone if $x \in S \Rightarrow \lambda x \in S$ for all $\lambda \geq 0$.

$$x_1, x_2 \in S \Rightarrow \lambda x_1 + (1-\lambda)x_2 \in S$$

for all $0 \leq \lambda \leq 1$.

Convex Hull! Let $S \subset \mathbb{R}^n$. Then the set of all convex combinations of sets of points of S is called a convex hull denoted by $C(S)$ or S^c of the set S .

The convex hull of a set is the intersection of all convex

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Sets containing S .

Example.

Let $S \subset \mathbb{R}^2$ be the set of points on the circle, then the convex hull of S by taking two points of S is the set of points in the circular region.

if x_1, x_2, x_3, x_4 and x_5 be given in \mathbb{R}^2 . Then the dotted lines represented the boundaries of the convex hull for these five points.

Convex Polyhedron: if the convex set S consists of a finite number of extreme points the convex hull of S is called a convex polyhedron.

if C is the convex polyhedron, then by definition

$$C = \left\{ x : x = C_1 x_1 + C_2 x_2 + C_3 x_3 + \dots + C_m x_m \right\}$$

where $C_i \geq 0$ for $i = 1, 2, 3, \dots, m$

$$C_1 + C_2 + C_3 + \dots + C_m = 1$$

and $x_1, x_2, x_3, \dots, x_m \in \mathbb{R}^n$

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Thus the convex polyhedron spanned by the two points x_1, x_2 is the set of points.

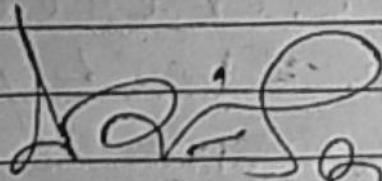
~~$x = c_1 x_1 + c_2 x_2$~~

$x = c_1 x_1 + c_2 x_2$
where $c_1, c_2 \geq 0$
and $c_1 + c_2 = 1$.

Simplex! 1. A Simplex is an n -dimensional convex polyhedron having exactly $(n+1)$ vertices.

2. in two dimensions, Simplex is a triangle.

3. in three dimensions, Simplex is a tetrahedron.


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