Ondated: 23/06/2024

Mathematics Honours Sub.

B. Sc. Part - I

Paper - I

Topic: LINEAR PROGRAMMING.

~~Example.~~

Old hens can be bought at Rs. 2 each and young ones Rs. 5 each. The old hens lay 3 eggs per week and the young ones lays 5 eggs per week. Each egg being worth 30 paise. A hen costs Rs. 1 per week to feed. I have only rupees 80.00 to spend for hens. How many of each kind should I buy to give a profit of more than Rs. 6 per week, assuming that I cannot house more than 20 hens.

Formulation: Let x_1 be the number of old hens and x_2 the number of young hens to be bought.

Since old hens lay 3 eggs per week and the young ones lay 5 eggs per week. Therefore the total eggs obtained per week

$= 3x_1 + 5x_2$

The total gain by the rate of 30 paise per egg

$=$ Rs. $0.30(3x_1 + 5x_2)$

The total expenditure for x_1+x_2 hens at the rate of Rs. 1 each

Thus the total profit z earned per week become $=$ Rs. 1. (x_1+x_2)

$Z =$ total gain $-$ total expenditure.

$= 0.30(3x_1 + 5x_2) - (x_1+x_2)$

$= 0.5x_2 - 0.1x_1$

Since old hens can be bought at Rs. 2 each and young ones at Rs. 5 each and there are only Rs. 80 available for purchasing the hens, therefore

$2x_1 + 5x_2 \leq 80.$

Also, Since it is not possible to house more than 20 hens at a time, therefore $x_1+x_2 \leq 20.$

Also Since the profit is restricted to be more than Rs. 6, this means that the profit function Z is to be maximized.

Thus, there is no need to add one more constraint $0.5x_2 - 0.1x_2 \geq 6$.

Again, it is not possible to purchase negative quantity of hens, therefore,
$x_1 \geq 0,\ x_2 \geq 0.$

Finally, the problem becomes: Find x_1 and x_2 so as to maximize the objective function.

$$Z = 0.50x_2 - 0.10x_1$$

Subject to the constraints

$$2x_1 + 5x_2 \leq 80$$
$$x_1 + x_2 \leq 20$$
$$x_1, x_2 \geq 0.$$

Graphical Solⁿ. Plot the straight lines $2x_1 + 5x_2 = 80$ and $x_1 + x_2 = 20$ on the graph and shade the feasible region as shown in the graph paper.

$\because 2x_1 + 5x_2 = 80$

$\Rightarrow 2x_1 = 80 - 5x_2$

$\therefore x_1 = \dfrac{80 - 5x_2}{2}$

if $x_2 = 2$ then $x_1 = 35$

$x_2 = 4$ then $x_1 = 30$

$x_2 = 6$ then $x_1 = 25$.

and $x_2 = -2$ then $x_1 = 45$

$x_2 = -4$ then $x_1 = 50$.

Table.

x_1	35	30	25	45	50
x_2	2	4	6	-2	-4

Eqn: $x_1 + x_2 = 20$

$\Rightarrow x_1 = 20 - x_2$

if $x_2 = 5$ then $x_1 = 15$

$x_2 = 10$ then $x_1 = 10$

$x_2 = 15$ then $x_1 = 5$

and $x_2 = -5$ then $x_1 = 25$

$x_2 = -10$ then $x_1 = 30$.

$x_2 = -15$ then $x_1 = 35$.

Table:

x_1	15	10	5	25	30	35
x_2	5	10	15	-5	-10	-15

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Date :........../........../..........

$$Z = 0.50x_2 - 0.10x_1.$$

Subject to the Constraints.

$$2x_1 + 5x_2 \leq 80$$
$$x_1 + x_2 \leq 20.$$
$$x_1, x_2 \geq 0.$$

Table! $2x_1 + 5x_2 = 80.$

x_1	35	30	25	45	50.
x_2	2	4	6	-2	-4

Table! $x_1 + x_2 = 20.$

x_1	15	10	5	25	30	35
x_2	5	10	15	-5	-10	-15

$2x_1 + 5x_2 = 80$ — (i) $\times 1$

$x_1 + x_2 = 20$ — (ii) $\times 5$

$\Rightarrow 2x_1 + \cancel{5x_2} = 80.$

$5x_1 + \cancel{5x_2} = 100$

$- \quad - \quad -$

$-3x_1 = -20$

$\therefore x_1 = \frac{-20}{-3} = 6.66,$

$\Rightarrow 6.7$

Put $x_1 = 6.7$ in eq^n (ii)

$x_1 + x_2 = 20$

$6.7 + x_2 = 20$

$\therefore x_2 = 20 - 6.7$

$= 13.3.$

$E(6.7, 13.3)$

The value of Z are the vertices are

$Z_0 = 0$

$Z_A = 0.50 \times 0 - 0.10 \times 16$

$= 0 - 1.6 = -1.6.$

$Z_B = 0.50 \times 20 - 0.10 \times 0$

$= 10 - 0 = 10.$

$Z_E = 0.50 \times 6.7 - 0.10 \times 13.3.$

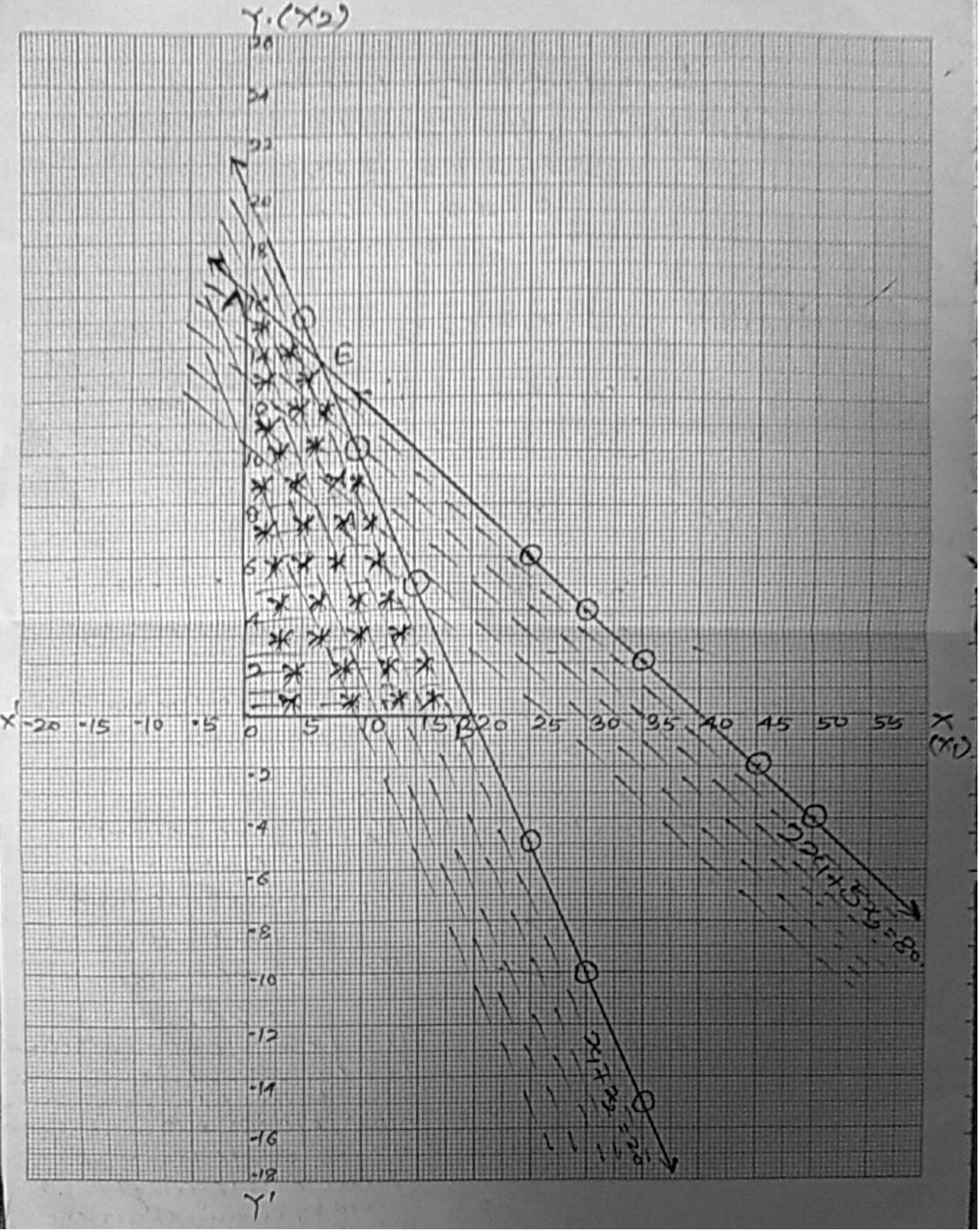
$= 2.02$

The feasible region is OBEA.

The co-ordinates of the extreme point of the feasible region $O(0,0)$, $A(0,16)$, $B(20,0)$, $E(6.7, 13.3)$

$\Rightarrow x_1 = 20$, $x_2 = 16$, max $Z = 10.$

Hence only 16 younghens. I should by in order to get the max. profit of Rs. 10 (>6).


Y.(X2)
X'
-20
-15
-10
-5
0
5
10
15
20
25
30
35
40
45
50
55
X
(X1)
E
B
2x1+5x2=80
x1+x2=20
Y'