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Mathematics Hons & Sub-
B.Sc. Part-I
Paper-I
Topic: Transportation Problem as A L.P.P.

Transportation problem is a special class of linear transportation problem. We substantiate this statement by taking an example.

Let us consider the three manufacturing plants O_1, O_2, O_3 and the product from the plants are transported to four destinations D_1, D_2, D_3 and D_4. The relevant data are recorded in the following table:

Destination →

Origins					Capacity
O_1	$x_{11}(c_{11})$	$x_{12}(c_{12})$	$x_{13}(c_{13})$	$x_{14}(c_{14})$	a_1
O_2	$x_{21}(c_{21})$	$x_{22}(c_{22})$	$x_{23}(c_{23})$	$x_{24}(c_{24})$	a_2
O_3	$x_{31}(c_{31})$	$x_{32}(c_{32})$	$x_{33}(c_{33})$	$x_{34}(c_{34})$	a_3.
Requirement	b_1	b_2	b_3	b_4	

As we know c_{ij} $(i=1,2,3,\ j=1,2,3,4)$ is the cost of transporting 1 unit of goods from the origin O_i $(i=1,2,3,)$

to the destination D_j ($j = 1, 2, 3, 4$) and the variable x_{ij} is the quantity to be transported from the origin O_i to the destination D_j

This transportation problem is like to the general L.P.P. based on three components.

(i) Here we have to minimise total transportation cost of all the goods transported from the origin to the destination.

minimise:

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{4} x_{ij} \cdot c_{ij},$$ which is the Linear objective function.

(ii) From the given table, we have the constraints.

Row wise constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} = a_1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = a_2$$
$$x_{31} + x_{32} + x_{33} + x_{34} = a_3.$$

These are called source or capacity constraints which specify the relationship between the origin capacities and the goods to be received by different destinations

Column wise constraints:

$x_{11} + x_{21} + x_{31} = b_1$

$x_{12} + x_{22} + x_{32} = b_2$

$x_{13} + x_{23} + x_{33} = b_3$

$x_{14} + x_{24} + x_{34} = b_4$

These are called demand or requirement constraints and the goods to be shipped from different origins.

(iii) The three component parts of our transportation problem are given as

Minimise: $Z = C_{11}x_{11} + C_{12}x_{12} + C_{13}x_{13} + C_{14}x_{14} + C_{21}x_{21} + C_{22}x_{22} + C_{23}x_{23} + C_{24}x_{24} + C_{31}x_{31} + C_{32}x_{32} + C_{33}x_{33} + C_{34}x_{34}$.

Subject to: $x_{11} + x_{12} + x_{13} + x_{14} = a_1$

$x_{21} + x_{22} + x_{23} + x_{24} = a_2$

$x_{31} + x_{32} + x_{33} + x_{34} = a_3$

$x_{11} + x_{21} + x_{31} = b_1$

$x_{12} + x_{22} + x_{32} = b_2$

$x_{13} + x_{23} + x_{33} = b_3$

$x_{14} + x_{24} + x_{34} = b_4$

and $x_{ij} \geq 0$, $i = 1, 2, 3$; $j = 1, 2, 3, 4$.

Thus the transportation problem is a special case of the general L.P. Problem.

Theorem: In the transportation problem

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \text{——} (1)$$

Subject to: $$\sum_{i=1}^{n} x_{ij} = \cancel{b_j}\ a_i\ ; \ i = 1,2,3 \ldots m \quad \text{——} (2)$$

$$\sum_{i=1}^{n} x_{ij} = b_j\ ; \ j = 1,2,3 \ldots n \quad \text{——} (3)$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ and } x_{ij} \geq 0 \quad \text{——} (4)$$

The number of non-degenerate basic feasible solution is at most $m+n-1$.

Proof:- We know that the transportation problem is a special case of linear programming problem with $(m+n)$ constraints. we are going to show that only $(m+n-1)$ of them are independent.

From (2), we have

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i \quad \text{——} (5)$$

From (3), we have

$$\sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n-1} b_j$$

i.e, $$\sum_{i=1}^{m} \sum_{j=1}^{n-1} x_{ij} = \sum_{j=1}^{n-1} b_j \qquad \text{——— (6)}$$

Substracting (6) from (5), we get

$$\sum_{i=1}^{m} \left[\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n-1} x_{ij} \right] = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n-1} b_j$$

$$= \sum_{i=1}^{n} b_j - \sum_{i=1}^{n-1} b_j = b_j$$

Hence $\sum_{i=1}^{m} x_{in} = b_n$; for $j = n$.

Thus we see that in a m-origin and n-destination transportation problem, which has got a feasible solution, we can state the problem completely with $(m+n-1)$ equations.

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