Date 29/06/2021

Mathematics Hons & Sub.
B. Sc. Part - I
Paper - I.
Topic: Assignment Problem (L.P.)

Solution of the Assignment Problem

Hungarian Method:

In solving ballanced assignment problem of n jobs and n machines the following computational steps are taken

Step I: To construct the starting table the smallest element in each row is substracted from every element in that row.

Step II: The smallest element in each column of this reduced cost matrix is substracted from every element in that column.

Step III: In the reduced cost matrix obtained in the Step II, we draw minimum number of vertical and horizontal lines to go across the zeroes of the starting table. Let this number be p

(i) if $p = n$, an Optimum assignment has been achieved.

(ii) if $p < n$, we go to the next Step.

Step IV: In the reduced matrix of III Step, we pick up the least cost in the Starting table among those through which p lines have not passed. We substract this least cost from all the uncovered points of the Starting table and add this least cost to all the points of the Starting table that lie at the intersection of horizontal and vertical lines. The resulting table is the second modified table. repeat the above Steps till we reach $p = n$.

Step V: We examine Rows one after another till we reach a Row with exactly one unmarked Zero. Put this Zero in a Square. The assignment will be made at this point. Put a Cross mark in the cells of all other Zeroes in the Column of the squared Zero.

The cross marked zero cannot be taken into account for other assignments.

Step VI: We follow the step V for columns also.

Step VII: We repeat the steps V and VI till no unmarked zeroes are left out. Now we have completed the assignment for optimum cost.

The assignment schedule indicated by squared zeroes in the optimum solution of the assignment problem.

EXAMPLES.

1. Solve the following assignment problem.

Subordinates.

		I	II	III	IV
	A	8	26	17	11
	B	13	28	4	26
Tasks.	C	38	19	18	15
	D	19	26	24	10

Soln. Step I

The Smallest entries in each row are 8, 4, 15, 10 respectively. Thus substracting the smallest eliment in each row from every element of the same row, we get the following matrix:

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Step II. The Smallest entries in each Column are 0, 4, 0, 0 respectively and thus substracting the smallest element in each Column in the reduced matrix from every element of the same Column. we get the following table

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Step III: we draw the minimum possible number of horizontal and vertical lines so as to cover all the zeroes. Thus we get the following table

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Since the number of lines is equal to the order of the cost matrix, an optimum assignment has been attained. For determining the optimum assignment, we consider only the zero elements of the matrix as given below

	I	II	III	IV
A	0			
B			0	
C		0		0
D				0

Since 1st, 2nd and 4th row contain single zero, we assign A to I, B to III and D to V.

The final assignment will be

	I	II	III	IV
A	$\boxed{0}$			
B			$\boxed{0}$	
C		$\boxed{0}$		0
D				$\boxed{0}$

In this table, every row and every column have one assignment. Thus the optimal zero assignment follows;

Tasks	A	B	C	D
Subordinates	I	III	II	IV
Man hour	8	4	19	10

The total man hour

$= 8+4+19+10$

$= \underline{41}$

Sd/-

Dr. Birendra, Asst. Prof.

Dept of Maths.

D.K. College,

Dumraon.