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Mathematics Hons & Sub.

B.Sc. Part - I

Paper - I

Topic: Theory of Equation.

Q: A root α of the eqn $3x^3 - 10x^2 + 7x + 10 = 0$ is connected with a root α' of the equation $x^3 - x^2 - 17x + 65 = 0$ by the relation $\alpha\alpha' + \alpha' - \alpha + 1 = 0$. Using the fact, solve the two given eqns completely.

Soln:- Since α is a root of the equation

$$3x^3 - 10x^2 + 7x + 10 = 0$$

we have, $3\alpha'^3 - 10\alpha'^2 + 7\alpha' + 10 = 0$ — (1)

Again since α' is a root of the equation $x^3 - x^2 - 17x + 65 = 0$, we have

$$\alpha'^3 - \alpha'^2 - 17\alpha' + 65 = 0 \quad \text{— (2)}$$

But $\alpha\alpha' + \alpha' - \alpha + 1 = 0$

i.e $\alpha'(\alpha + 1) = \alpha - 1$

$$\therefore \alpha' = \frac{\alpha - 1}{\alpha + 1}$$

Now, putting $\alpha' = \frac{\alpha - 1}{\alpha + 1}$ in (2)

we get, $\left(\frac{\alpha - 1}{\alpha + 1}\right)^3 - \left(\frac{\alpha - 1}{\alpha + 1}\right)^2 - 17\left(\frac{\alpha - 1}{\alpha + 1}\right) + 65 = 0$

$$\Rightarrow (\alpha-1)^3 - (\alpha-1)^2(\alpha-1) - 17(\alpha-1)(\alpha+1)^2 + 65(\alpha+1)^3 = 0.$$

$$\Rightarrow (\alpha^3 - 3\alpha^2 + 3\alpha - 1) - (\alpha^3 - \alpha^2 - \alpha + 1) - 17(\alpha^2-1)(\alpha+1) + 65(\alpha^3 + 3\alpha^2 + 3\alpha + 1) = 0$$

$$\Rightarrow (\alpha^3 - 3\alpha^2 + 3\alpha - 1) - (\alpha^3 - \alpha^2 - \alpha + 1) - 17(\alpha^3 + \alpha^2 - \alpha - 1) + 65(\alpha^3 + 3\alpha^2 + 3\alpha + 1) = 0.$$

$$\Rightarrow \alpha^3(1-1-17+65) + \alpha^2(-3+1-17+195) + \alpha(3+1+17+195) + (-1-1+17+65) = 0.$$

$$\Rightarrow 48\alpha^3 + 176\alpha^2 + 216\alpha + 80 = 0$$

$$\Rightarrow 6\alpha^3 + 22\alpha^2 + 27\alpha + 10 = 0 \quad \text{— (3)}$$

Hence the roots of $3x^3 - 10x^2 + 7x + 10 = 0$.

are $-\frac{2}{3}$, $2+i$, $2-i$.

Again, dividing

$x^3 - x^2 - 17x + 65$ by $x+5$, we get

$x^2 - 6x + 13$

Solving $x^2 - 6x + 13 = 0$, we get

$$x = \frac{6 \pm \sqrt{36 - 4\cdot1\cdot13}}{2}$$

$$= \frac{6 \pm 4i}{2} = 3 \pm 2i$$

Hence the roots of $x^3 - x^2 - 17x + 65 =$

are -5, $3+2i$, $3-2i$.

Q: if the root of the equation $x^4 - 5x^3 + 11x^2 - 13x + 6 = 0$ be connected by the relation $2\beta + 3\alpha = 7$, then find all its roots.

Solⁿ: Putting $x = 1$ in $x^4 - 5x^3 + 11x^2 - 13x + 6$, we find that $x^4 - 5x^3 + 11x^2 - 13x + 6 = 0$.

taking $\alpha = 1$, in the relation $2\beta + 3\alpha = 7$, we get

$$2\beta = 7 - 3 = 4, \therefore \beta = 2$$

Hence the two roots of the given equation are $\alpha = 1$ and $\beta = 2$

Consequently $(x-1)$ and $(x-2)$ are two factor of $x^4 - 5x^3 + 11x^2 - 13x + 6$

Now, dividing

$x^4 - 5x^3 + 11x^2 - 13x + 6$ by $(x-1)(x-2)$

i.e, by $x^2 - 3x + 2$, we get $x^2 - 2x + 3$.

$\therefore$ The other roots of the given eqⁿ are obtained by the equation $x^2 - 2x + 3 = 0$. Solving $x^2 - 2x + 3 = 0$

we get, $x = \dfrac{2 \pm \sqrt{4 - 4.1.3}}{2} = \dfrac{2 \pm 2\sqrt{2}i}{2}$

$$= 1 \pm \sqrt{2}i$$

Hence all the four roots of the given eqⁿ are $1, 2, 1+\sqrt{2}i, 1-\sqrt{2}i$

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