

05.07.2021.

Mathematics Honors of Sub  
B. Sc. Part - I.  
Paper - I

Topic: Relation between the  
roots and coefficients  
(Exp. based on the Theory of eqs)

Q: Show that if the equation  
 $x^3 - ax^2 + bx - c = 0$  has pair  
of roots of the form  $d(1 \pm i)$ ,  
where  $d$  is real, then

$$(a^2 - 2b)(b^2 - 2ca) = c^2$$

Hence find the roots of the  
equation  $x^3 - 7x^2 + 20x - 24 = 0$

Sol: Let the third root of  
the given equation be  $\beta$ .

Now,

$$\begin{aligned}\sum \alpha &= (d+id) + (d-id) + \beta \\ &= 2d + \beta = a \quad \text{--- (1)}\end{aligned}$$

$$\begin{aligned}\sum \alpha\beta &= (d+id)(d-id) + \\ &\beta(d+id) + \beta(d-id) = b \quad \text{--- (2)}\end{aligned}$$

$$\text{Or, } d^2 + d^2 + 2d\beta = b$$

$$\text{Or, } 2(d^2 + d\beta) = b \quad \text{--- (2)}$$

Equaling (1) we get

$$4d^2 + 4d\beta + \beta^2 = a^2 \quad \text{--- (3)}$$

Multiplying (1) by 2 and  
Subtracting it from (3), we get

$$\beta^2 = d^2 - 2\beta \quad \text{--- (A)}$$

Also as  $\beta$  is a root of the  
given equation, so we have

$$\beta^2 - 2\beta^2 + b\beta - c = 0.$$

$$\text{Or, } \beta(\beta^2 + b) - d\beta^2 = 0.$$

$$\text{Or, } \beta(a^2 - 2b) + b - a(a^2 - 2b) - c = 0$$

From (A)

$$\text{Or, } (a^2 - 2b)(b^2 - 2ac) = c^2 \quad \underline{\underline{\text{proved}}}$$

Now, we are to solve the eqn

$$x^3 - 7x^2 + 20x - 24 = 0.$$

$$\text{Here } a = 7, b = 20, c = 24$$

From (A), we get

$$2d + \beta = a$$

$$\text{Or, } 2d + 3 = 7$$

$$\text{Or, } d = 2$$

∴ The required roots  
are  $d(1 \pm i), \beta$ .

$$\text{Where } d = 2, \beta = 3$$

$$\text{i.e., } 2(1 \pm i), 3.$$

Q. Show that the condition, that the cubic equation  $x^3 + px^2 + qx + r = 0$  should have two roots  $\alpha, \beta$  connected by the relation  $\alpha\beta + 1 = 0$  is  $1 + q + pr + r^2 = 0$ .

Sol<sup>n</sup>: - The given cubic equation  $x^3 + px^2 + qx + r = 0$

Let  $\gamma$  be the third root of this equation,

then  $\alpha\beta\gamma = -r$

$$\text{or, } (-1)\gamma = -r, \because \alpha\beta = -1$$

$$\therefore \gamma = r.$$

But  $\gamma$  is a root of the given equation

$$\therefore \gamma^2 + p\gamma + q + \frac{r}{\gamma} = 0 \quad \text{--- (1)}$$

putting  $\gamma = r$  in (1), we get

$$r^2 + pr + q + \frac{r}{r} = 0$$

$$\text{or, } r^2 + pr + q + 1 = 0$$

which is the required condition.

Q. Find the condition when the roots of the equation  $x^4 + px^3 + qx^2 + rx + s = 0$  are connected by the relation  $\alpha\beta = -1$ , where  $\alpha, \beta, \gamma, \delta$  are

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the roots of the given equation.

Sol<sup>n</sup>:-

Since  $\alpha, \beta, \gamma, \delta$  are the roots of the given equation  $x^4 + px^3 + qx^2 + rx + S = 0$ ,

$$\therefore \alpha\beta\gamma\delta = -S$$

$$\therefore \gamma\delta = -S, \quad \because \alpha\beta = -1$$

Now, we can write

$$x^4 + px^3 + qx^2 + rx + S$$

$$= (x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$$

$$= (x - ax + \alpha\beta)(x^2 - bx + \gamma\delta)$$

where  $a = \alpha + \beta$ ,  $b = \gamma + \delta$ .

$$\text{Or, } x^4 + px^3 + qx^2 + rx + S$$

$$= (x^2 - ax - 1)(x^2 - bx - S)$$

$$\therefore \alpha\beta = -1, \gamma\delta = -S.$$

Comparing the coefficients of the powers of  $x$  on both sides, we get

$$p = -(a+b) \quad \text{--- (1)}$$

$$q = -S - 1 + ab \quad \text{--- (2)}$$

$$\text{and } r = b + aS \quad \text{--- (3)}$$



Adding (1) & (2) we get,

$$p+r = a(s-1)$$

$$\text{or } a = \frac{p+r}{s-1}$$

From (1), we get,

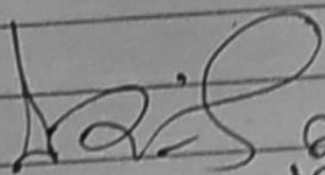
$$b = -a - p = -\frac{p+r}{s-1} - p$$
$$= -\frac{ps+r}{s-1}$$

Now substituting the value of  $a$  and  $b$  in (2), we get

$$q+s+1 = ab = \left(\frac{p+r}{s-1}\right) \left(-\frac{ps+r}{s-1}\right)$$

$$\text{Or, } (q+s+1)(s-1)^2 = -(ps+r)(p+r)$$

$$\text{Or, } s^2 + s^2(q-1) + s(p^2+pr-2q-1) + (1+q+pr+r^2) = 0$$



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