06.07.2021.

Mathematics Hons.
B. Sc. Part-I
Paper-I

Topic. Exp based on roots & co-efficients relation (Theory of equation)

(1) Find the condition that the equation $x^4+px^3+qx^2+rx+S=0$ may have all its roots equal.

Solⁿ:- Let α be each root of the given equation $x^4+px^3+qx^2+rx+S=0$.

$\therefore \alpha+\alpha+\alpha+\alpha = -p$

Or, $4\alpha = -p$. ——— (1)

$\therefore \alpha = -\frac{1}{4}p$ ——— (2)

$\Sigma \alpha.\alpha = 6\alpha^2 = q$

Or, $\alpha^2 = \frac{1}{6}q$ ——— (2)

$\Sigma \alpha.\alpha.\alpha = 4\alpha^3 = -r,$

$\therefore \alpha^3 = -\frac{1}{4} r$ ——— (3)

and $\alpha \cdot \alpha \cdot \alpha \cdot \alpha = \alpha^4 = s$ — (4)

from (1) and (2), we get

$\left(-\frac{1}{4} p\right)^2 = \frac{1}{6} q$

Or, $3p^2 = 8q$ ——— (5)

from (1) and (3), we get

$\left(-\frac{1}{4} p\right)^3 = -\frac{1}{4} r$

Or $p^3 = 16r$ ——— (6)

and from (1) and (4), we get

$\left(-\frac{1}{4} p\right)^4 = s$, Or $p^4 = 256s$ — (7)

Thus the required conditions are given by relation (5), (6) and (7).

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(2) if $x^3+3px^2+3qx+r=0$ and $x^2+2px+q=0$ have a common factor, show that $4(p^2-q)(q^2-pr)=(pq-r)^2$.

Soln:- if $(x-\alpha)$ be a common factor, then α is a common root of both the equations.

$\therefore \alpha^3+3p\alpha^2+3p\alpha+r=0$ — (1)

and $\alpha^2+2p\alpha+q=0$ — (2)

Multiplying (2) by α, we get,

$$\alpha^3+2p\cdot\alpha^2+q\cdot\alpha=0 \quad \text{— (3)}$$

Now, subtracting equation (3) from (1), we get

$p\alpha^2+2p\alpha+r=0$ — (4)

Solving (2) and (4), we get

$$\frac{\alpha^2}{2pr-2q^2}=\frac{\alpha}{pq-r}=\frac{1}{2q-2p^2}$$

$$\therefore \alpha=\frac{2pr-2q^2}{pq-r}$$

and $\alpha = \frac{pq-r}{2q-2p^2}$

$\therefore \frac{2pr-2q^2}{pq-r} = \frac{pq-r}{2q-2p^2}$

Or, $(pq-r)^2 = (2pr-2q^2)(2q-2p^2)$

Or, $(pq-r)^2 = 4(pr-q^2)(q-p^2)$

Or, $(pq-r)^2 = 4(p^2-q)(q^2-pr)$

Hence the result.

(8) If $f(x)=0$ is a cubic equation whose roots are α, β, γ and α is the harmonic mean of the roots of $f'(x)=0$, show that $\alpha^2 = \beta\gamma$.

Solⁿ:- Let $f(x) = x^3 + px^2 + qx + r = 0$ — (1)

then $f'(x) = 3x^2 + 2px + q = 0$ — (2)

$\because \alpha, \beta, \gamma$ are the roots of (1)

$\therefore \alpha+\beta+\gamma = -p$

$\Sigma \alpha\beta = q$

$\alpha\beta\gamma = -r.$

Again, let x_1 and x_2 be the roots of (2), then

$$x_1+x_2 = -\frac{2p}{3}$$

and $x_1 x_2 = \frac{q}{3}$.

If α is the harmonic mean of x_1, x_2, then x_1, α, x_2 are in H.P.

$$\therefore \alpha = \frac{2x_1x_2}{x_1+x_2} = \frac{2\cdot\frac{q}{3}}{-\frac{2p}{3}} = -\frac{q}{p} \quad \text{--- (3)}$$

Now, $\dfrac{\Sigma\alpha\beta}{\Sigma\alpha} = \dfrac{q}{-p}$ --- (4)

From (3) and (4), we get

$$\alpha = \frac{\Sigma\alpha\beta}{\Sigma\alpha} = \frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha+\beta+\gamma}.$$

Or, $\alpha(\alpha+\beta+\gamma) = \alpha\beta+\beta\gamma+\alpha\gamma$.

Or, $\alpha^2+\alpha\beta+\alpha\gamma = \alpha\beta+\beta\gamma+\alpha\gamma$.

Or, $\alpha^2 = \beta\gamma$ $\therefore$ Hence the result

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