Mathematics

B.Sc. (Maths Hons)

Part-I

Paper-I

Name of the Topic:

Trigonometry (Hyperbolic function)

Defn:- Whether z be real or complex, the hyperbolic sine and cosine of z (written as $\sinh z$, $\cosh z$ respectively) are defined as follows:

$$\sinh z = \frac{e^{z} - e^{-z}}{2}$$

$$\cosh z = \frac{e^{z} + e^{-z}}{2}$$

The hyperbolic tangent, cosecant and cotangent are obtained from the hyperbolic sine and cosine just as the circular tangent, cosecant, secant and cotangent are obtained from the circular sine and cosine.

Thus $\tanh z = \dfrac{\sinh z}{\cosh z}$

$$\Rightarrow \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}.$$

$$\operatorname{cosech} z = \frac{1}{\sinh z} = \frac{2}{e^{z} - e^{-z}}$$

$$\operatorname{sech} z = \frac{1}{\cosh z} = \frac{2}{e^{z} + e^{-z}}$$

$$\coth z = \frac{1}{\tanh z} = \frac{e^{z} + e^{-z}}{e^{z} - e^{-z}}$$

Relation between Circular functions and Hyperbolic functions:

For all values of θ, we have

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

and $$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

Hence putting ix for θ, it follows that

$$\sin ix = \frac{1}{2i}\left(e^{i\cdot ix} - e^{-i\cdot ix}\right)$$

$$= \frac{1}{2i}\left(e^{-x} - e^{x}\right)$$

$$= -\frac{1}{2i}\left(e^{x} - e^{-x}\right)$$

$$= \frac{i}{2}\left(e^{x} - e^{-x}\right) = i\sinh x.$$

and $\cos ix = \frac{1}{2}(e^{i.ix} + e^{-i.(ix)})$

$= \frac{1}{2}(e^{-x} + e^{x}) = \frac{1}{2}(e^{x} + e^{-x})$

$= \cosh x.$

Thus,

$$\left.\begin{aligned} \sin ix &= i \sinh x, \\ \cos ix &= \cosh x \\ \tan ix &= \frac{\sin ix}{\cos ix} \\ &= i \tanh x \text{ etc.} \end{aligned}\right\} \quad (1)$$

and conversely,

$$\left.\begin{aligned} \sinh x &= \frac{1}{i} \sin ix \\ &= -i \sin ix \\ \cosh x &= \cos ix \\ \text{and } \tanh x &= -i \tan ix \text{ etc} \end{aligned}\right\} \quad (2)$$

in general corresponding to most trigonometrical formulae involving the circular functions there are formulae involving the hyperbolic functions.

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