Mathematics Hons

B. Sc. Part - II

Paper - IV

Topic: Linear equations with Constant Coefficients (Diff. eqn)

Linear equations are those equations in which dependent variable and its derivatives appear only in the first degree and are not multiplied together.

The general differential equation of the n^{th} order is

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = X.$$

Where X and the coefficients $P_1, P_2, P_3 --- P_n$ are functions of x only.

Here the equation is linear since the dependent variable y and its differential coefficients occur in the first degree. if $P_1, P_2, P_3 --- P_n$ are constants and X be the function of x alone, then the above equation is said to be the linear differential equation with constants coefficients.

Linear Equations with Constant Coefficients:

Let the general linear equation with constant coefficients be of the form

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}}$$

$$+ \cdots + P_n y = X \quad (1)$$

where $P_1, P_2, \cdots P_n$ are constants and X is a function of x for a constant.

Theorem I:-

If y_1, y_2 are any two solution of

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}}$$

$$+ \cdots + P_n y = 0 \quad (2)$$

(The R.H.S, $x=0$), then

$c_1y_1 + c_2y_2$ is also a solution of (2); c_1, c_2 being arbitrary constants.

Proof:- Since y_1, y_2 are solution of (2), we have

$$\frac{d^n y_1}{dx^n} + P_1\frac{d^{n-1}y_1}{dx^{n-1}} + P_2\frac{d^{n-2}y_1}{dx^{n-2}} + \cdots + P_n y_1 = 0 \quad (3)$$

$$\frac{d^n y_2}{dx^n} + P_1\frac{d^{n-1}y_2}{dx^{n-1}} + P_2\frac{d^{n-2}y_2}{dx^{n-2}} + \cdots + P_n y_2 = 0 \quad (4)$$

In order to show that $c_1y_1 + c_2y_2$ is a solution of (2), we need to show that $y = c_1y_1 + c_2y_2$ satisfies (2)

Now, $\frac{d^n}{dx^n}(c_1y_1 + c_2y_2) + P_1\frac{d^{n-1}}{dx^{n-1}}(c_1y_1 + c_2y_2) + P_2\frac{d^{n-2}}{dx^{n-2}}(c_1y_1 + c_2y_2) + - - -$

$$= C_1\left(\frac{d^n y_1}{dx^n} + P_1\frac{d^{n-1}y_1}{dx^{n-1}} + P_2\frac{d^{n-2}y_1}{dx^{n-2}} + \cdots\right)$$

$$+ C_2\left(\frac{d^n y_2}{dx^n} + P_1\frac{d^{n-1}y_2}{dx^{n-1}} + P_2\frac{d^{n-2}y_2}{dx^{n-2}} + \cdots\right)$$

$$= C_1 \times 0 + C_2 \times 0$$

(because of (3) and (4))

$$= 0.$$

This proves the theorem.

Symbolic Representation of

$$\frac{d^n y}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + P_n y = 0$$

Let, D Stands for $\frac{d}{dx}$

D^2 for $\frac{d^2}{dx^2}$,...

and D^n for $\frac{d^n}{dx^n}$.

Dr. Birendra Prasad
Asst. Prof.
Dept. of Mathematics
D.K. College,
Dumraon