Mathematics Hons.
B. Sc. Part - II
Paper - IV
Topic: Particular Integrals
(Diff - eqns)

The methods of obtaining particular Integrals of non-homogeneous partial differential equations are very similar to those used in solving linear equations with constant coefficients. Here we are considering few cases.

CASE I :- When the R.H.S. of differential equation is of the form e^{ax+by}.

Then $\frac{1}{f(D,D')} e^{ax+by} = \frac{1}{f(a,b)} e^{ax+by}$ provided $f(a,b) \neq 0$

i.e, put $D = a$ and $D' = b$.

CASE II :- When the R.H.S of the differential equation is of the form $\sin(ax+by)$ or $\cos(ax+by)$.

Then $\frac{1}{f(D,D')}$ Sin (ax+by)

or Cos (ax+by) is obtained by putting $D^2 = -a^2$, $DD' = -ab$

$D'^2 = -b^2$ provided the denominator is not zero.

CASE III : When the R.H.S of the differential equation is of the form $x^m y^n$ where m and n are positive integers then,

$$\frac{1}{f(D,D')} x^m y^n = [f(D,D')]^{-1} x^m y^n$$

CASE IV : When the R.H.S of the differential equation is of the form

$$e^{ax+by} V.$$

Then,

$$\frac{1}{f(D,D')} e^{ax+by} V$$

$$= e^{ax+by} \frac{1}{f(D+a, D'+b)} V$$

Some Examples.

(1) Solve: $(D^2 - D'^2 + D - D')z = 0$

Solⁿ: When factorised

$D^2 - D'^2 + D - D'$

$= (D^2 - D'^2) + (D - D')$

$= (D - D')(D + D')(D - D')$

$= (D - D')(D + D' + 1)$

Hence the given equation is

$(D - D')(D + D' + 1)z = 0.$

$\therefore z = f_1(y + x) + e^{-x} f_2(y - x)$

(2) Solve: $DD'(D - 2D' - 3)z = 0$

Solⁿ:- The given equation can be written as

$(D - 0 \cdot D' + 0)(0 + D' - 0)(D - 2D' - 3)z = 0$

$\therefore z = e^{0 \cdot x} f_1(y + 0 \cdot x) + e^{0 \cdot x} f_2(0 \cdot x) + e^{3x} f_3(y + 2x)$

$= f_1(y) + \phi(x) + e^{3x} f_3(y + 2x)$

(3) Solve: $r + 2s + t + 2p + 2q + z = 0$

Solⁿ:- The above equation can be written as

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} + z = 0.$$

$$\Rightarrow (D^2 + 2DD' + D'^2 + 2D + 2D' + 1)z = 0.$$

$$\Rightarrow (D + D' + 1)^2 z = 0.$$

Hence $z = e^{-x} f_1(y-x) + x e^{-x} f_2(y-x)$

(4) Solve: $(2D^4 - 3D^2D' + D'^2)z = 0$

Soln!

The given equation can be written as

$$(2D^2 - D')(D^2 - D')z = 0.$$

However, non of factors can be further resolved in to factors linear in D and D'.

Hence let $z = Ae^{hx+ky}$ be the C.F. corresponding to

$$(D^2 - D')z = 0$$

$$\Rightarrow (D^2 - D')z = Ah^2 e^{hx+ky} - Ak e^{hx+ky}$$

$$= A(h^2 - k)e^{hx+ky} \quad \text{(1)}$$

∴ The given equation (1) will

be satisfied by the substitution if $h^2 - k = 0$.

$\therefore$ The C.F corresponding to $(D^2 - D')z = 0$ is $\sum A e^{hx + h^2 y}$.

Similarly C.F corresponding to $(2D^2 - D')z = 0$ is

$$\sum B e^{h'x + k'y}$$

Where $2h'^2 - k' = 0$ i.e

$$\sum B e^{h'x + 2h'^2 y}.$$

$\therefore$ The most general solution of the given equation is

$$z = \sum A e^{hx + h^2 y} + \sum B e^{h'x + 2h'^2 y}$$

(5) Solve $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$.

Soln: The C.F corresponding to the first factor is $z = e^x f_1(y + 2x)$

The C.F corresponding to the 2nd factor is $\sum A e^{hx + ky}$.

Where $h - 2k^2 - 1 = 0$ i.e, $h = 2k^2 + 1$

Hence $z = e^x f_1(y + 2x) + \sum A e^{hx + ky}$ where $h = 2k^2 + 1$.

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