Mathematics Hons & Sub.
B.Sc. Part-I.
Paper-I
Topic: Relation between the roots and co-efficients.
(Theory of Equation)

Q. (1.) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$, two of whose roots are in the ratio (i) 2 : 3 (ii) 3 : 2.

Solⁿ :- Let the roots of the equation be $2\alpha$, $3\alpha$ and $\beta$. Then we have

$2\alpha + 3\alpha + \beta = 9$ or, $5\alpha + \beta = 9$ — (1)

$2\alpha \cdot 3\alpha + 2\alpha \cdot \beta + 3\alpha \cdot \beta = 14$

or, $6\alpha^2 + 5\alpha\beta = 14$ — (2)

and $2\alpha \cdot 3\alpha \cdot \beta = -24$

or, $6\alpha^2\beta = -24$

or $\alpha^2\beta = -4$ — (3)

Multiplying (1) by $5\alpha$, we get

$25\alpha^2 + 5\alpha\beta = 45\alpha$ — (4)

Substracting (2) from (4), we get

$19\alpha^2 - 45\alpha + 14 = 0$ — (5)

Solving (5), we get

$\alpha = \dfrac{45 \pm \sqrt{(45)^2 - 4 \cdot 19 \cdot 14}}{38}$

$= \dfrac{45 \pm \sqrt{2025 - 1064}}{38}$

$= \dfrac{45 \pm 31}{38}$

$= \dfrac{76}{38}$ or $\dfrac{14}{38}$

$= 2$ or $\dfrac{7}{19}$

if $\alpha = 2$, then from (i'), $\beta = 9 - 10 = -1$

if $\alpha = \dfrac{7}{19}$ the from (i'), $\beta = 9 - 5\alpha$

$= 9 - \dfrac{5 \cdot 7}{19} = 9 - \dfrac{35}{19}$

$= \dfrac{136}{19}$

But $\alpha = \dfrac{7}{19}$ and $\beta = \dfrac{136}{19}$ do not satisfy (3),

Hence we take $\alpha = 2$

Therefore the roots are 4, 6, -1.

(ii) Let the roots of the given equation be $3\alpha$, $2\alpha$, $\beta$.

Then $\Sigma\alpha = 3\alpha + 2\alpha + \beta = 9$

Or, $5\alpha + \beta = 9$ ——— (1)

$\Sigma\alpha\beta = 3\alpha.\beta + 2\alpha.\beta + 3\alpha.2\alpha$
$= 14$

Or, $5\alpha\beta + 6\alpha^2 = 14$ — (2)

and $3\alpha . 2\alpha . \beta = -24$

Or, $\alpha^2\beta = -4$ — (3)

from (1) and (2) we get,

$5\alpha(9-5\alpha) + 6\alpha^2 = 14$

$\because \beta = 9 - 5\alpha$ from (1)

Or, $19\alpha^2 - 45\alpha + 14 = 0$

Or, $19\alpha^2 - 38\alpha - 7\alpha + 14 = 0$

Or, $(19\alpha - 7)(\alpha - 2) = 0$

Or, $\alpha = 2, \frac{7}{19}$

$\therefore$ From (1), $\beta = -1$ Or $\frac{136}{19}$ as

$\alpha = 2$ Or $\frac{7}{19}$

Also (2) satisfied only for $\alpha = 2$ and $\beta = -1$. Hence the required roots are 6, 4, -1.

Q. (2) if One root of the equation $x^3 - px^2 + qx - r = 0$ be n times the other root of the equation show that it may be found from a quadratic.

Sol$^n$:-

Let the roots of the equation $x^3 - px^2 + qx - r = 0$ are $\alpha, n\alpha$ and $\beta$. Then

$\Sigma\alpha = \alpha + n\alpha + \beta = p$ —— (1)

$\Sigma\alpha\beta = \alpha \cdot n\alpha + \alpha \cdot \beta + n\alpha \cdot \beta = q$

Or, $n\alpha^2 + (n+1)\alpha\beta = q$

From (1), $\beta = p - (n+1)\alpha$ —— (2)

Substituting this value of $\beta$ in (2), we get the required quadratic as

$n\alpha^2 + (n+1)\alpha[p - (n+1)\alpha] = q$

Or $[n - (n+1)^2]\alpha + (n+1)p\alpha - q = 0$

Or, $(n^2+n+1)\alpha^2 - (n+1)p\alpha + q = 0$

(3) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in a arithmetical progression and hence solve $x^3 - 12x^2 + 39x = 28$.

Soln.:-

Let the roots of the equation be

$a-d,\ a,\ a+d$

$\therefore$ Sum of the roots $=$

$(a-d) + a + (a+d) = p$

Or $3a = p$ —— (1)

Since a is a root of the given equation, so we have $a^3 - pa^2 + qa - r = 0$

Or, $\frac{1}{27}p^3 - p.\frac{1}{9}p^2 + q.\frac{p}{3} - r = 0$ from (1)

Or, $2p^3 - 9pq + 27r = 0$

which is the required condition.

Now, Let $a-d$, $a$, $a+d$ be the roots of $x^3 - 12x^2 + 39x - 28 = 0$

$\therefore \Sigma\alpha = (a-d) + a + (a+d) = 12$

Or, $3a = 12$ or $a = 4$ — (1)

$\Sigma\alpha\beta = (a-d) + a + (a-d)(a+d) + a(a+d) = 39$

Or, $3a^2 - d^2 = 39$ Or, $48 - d^2 = 39$ ($\because a = 4$)

Or, $d^2 = 9$, $\therefore d = \pm 3$.

$\therefore$ The roots are: 1, 4, 7.

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