Mathematics Hons. Sub.
B. Sc. Part - I
Paper - I.
Topic: Linear Programming
(Convex set)

Convex Set: A subset A of the Euclidean space R^n is called a convex set if for every pair $x_1, x_2 \in A$, the line segment
$V = \lambda x_1 + (1-\lambda)x_2$, where $0 \le \lambda \le 1$
lies in A. i.e, $V \in A$.
This can be written as
$x_1, x_2 \in A \Rightarrow \lambda_1 x_1 + \lambda_2 x_2 \in A$.
where $\lambda_1, \lambda_2 \ge 0$ and $\lambda_1 + \lambda_2 = 1$.
Thus A is convex set if convex combination of every pair of points of x is also a points of A.

Example:

(1) The set of points in the interior of a circle.

(2) The set of points of the two dimensional plane R^2

Extreme points of a Convex Set:

x is the extreme point of the convex set A, if it is impossible to find $y, z\ (\neq x) \in A$, such that
$x = \lambda y + (1-\lambda) z;\ 0 < \lambda < 1.$

The vertices of a triangle are the only three extreme points of the convex set of points on the triangle and its interior and every point on the boundary of a circle is an extreme point of the convex set.

Convex Combination:

Let $x_1, x_2, x_3, \ldots x_m$ be m points in R^n and $x \in R^n$ such that

$$x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \ldots + \lambda_m x_m$$

$$= \sum_{i=1}^{m} \lambda_i x_i$$

where the λ_i's are scalars, is called a convex combination of the given vector. if

$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_m \geq 0$

and $\sum_{i=1}^{m} \lambda_i = 1.$

Thus a line segment $[x : x = \lambda_1 x_1 + \lambda_2 x_2 ; \lambda_1, \lambda_2 \geq 0$ and $\lambda_1 + \lambda_2 = 1]$ Joining two points x_1 and x_2 is the set of all convex combinations of x_1 and x_2.

Convex Cone: A non-empty set $S \subset R^n$ is called a cone if for each x in S and $\lambda \geq 0$, the vector λx is also in S.

A cone is called a convex cone if it is a convex set. Thus S is a convex cone if $x \in S \Rightarrow \lambda x \in S$ for all $\lambda \geq 0$.

$x_1, x_2 \in S \Rightarrow \lambda x_1 + (1-\lambda) x_2 \in S$, for all $0 \leq \lambda \leq 1$.

Convex Hull: Let $S \subset R^n$, then the set of all convex combinations of sets of points of S is called a convex hull denoted by $C(S)$ or S^c of the set S.

The convex hull of a set is the intersection of all convex sets containing S.

Example.

Let $S \subseteq R^2$ be the set of points on the circle, then the convex hull of S by taking two points of S is the set of points in the circular region.

If x_1, x_2, x_3, x_4 and x_5 be given in R^2. Then the dotted lines represented the boundaries of the convex hull for these five points.

Convex Polyhedron: If the convex set S consists of a finite number of extreme points the convex hull of S is called a convex polyhedron.

If C is the convex polyhedron, then by definition

$$C = \{x : x = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_mx_m\}$$

where $c_i \geq 0$ for $i = 1, 2, 3, \ldots m$

$$c_1 + c_2 + c_3 + \cdots + c_m = 1$$

and $x_1, x_2, x_3, \ldots x_n \in R^n$

Thus the convex polyhedron spanned by the two points x_1, x_2 is the set of points.

$$x = c_1x_1 + c_2x_2$$

where $c_1, c_2 \geq 0$

and $c_1 + c_2 = 1$.

Simplex: 1. A simplex is an n-dimensional convex polyhedron having exactly $(n+1)$ vertices.

2. In two dimensions, simplex is a triangle.

3. In three dimensions, simplex is a tetrahedron.

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