Mathematics Hons.
B.Sc. Part-II
paper-IV
Topic: System Of Coplanar forces (Statics)

Force| Force is an external effort in the form of push or pull which (i) Produces motion in a body (ii) Stops a moving body (iii) changes the direction of motion of the body.

Force (P) = mass x acceleration
$P = m f$.

System of Coplanar forces:

The System of coplanar forces acting on a rigid body is reduced to a single force R acting at an arbitrary chosen point 'O' in the plane of the forces together with the couple say G.

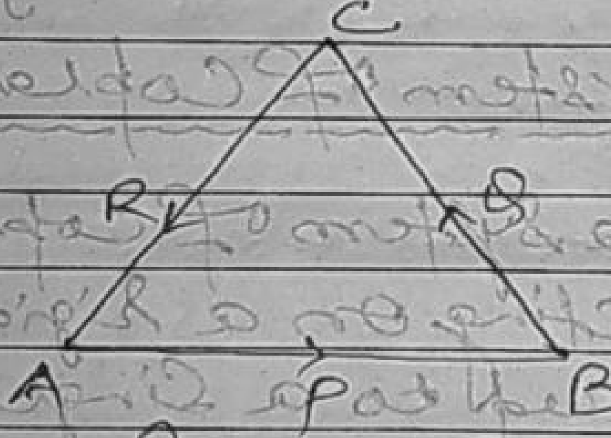
Vector:- The Physical quantities which have both magnitude and direction are called vector.

e.g. velocity, momentum, acceleration, force etc.

Coplanar vectors: The vectors which are acting in the same plane are called coplanar vectors.

The Triangle law of forces:

If three forces acting on a particle at the same time are represented in magnitude and direction by the sides of a triangle taken in one order then the particle will be in equilibrium.



From figure $P + Q + R = 0$.

Polygon of forces: If number of forces acting on a particle at the same time are represented in magnitude and direction by the sides of closed polygon taken in one order, then the forces will be in equilibrium. These forces are called polygon of forces.

From figure:

$$P + Q + R + S + T = 0.$$

Couple: When two equal unlike parallel forces acting on a body then this forces are called couple.

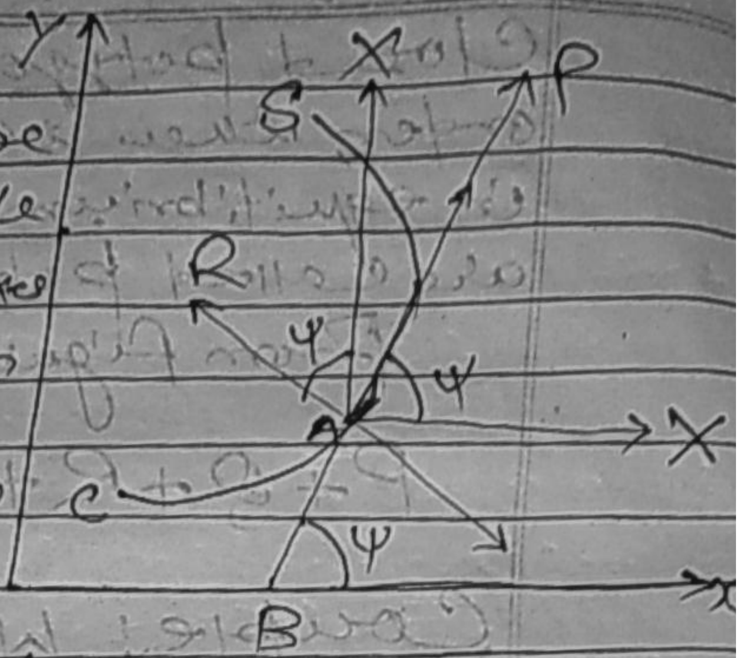
By the action of couple force we can twist the body round without moving in any direction.

Theorem: If a particle rest on a smooth curve under the action of any force, find the position of equilibrium.

Soln:

Let $y = f(x)$ be the equation of the smooth curve $A(x, y)$ be any position of the particle.

Let ψ be the angle which makes between tangent and x-axis. Then,



$\sin\psi = \frac{dy}{ds}$, $\cos\psi = \frac{dx}{ds}$

and $\tan\psi = \frac{dy}{dx}$ —— (1)

where CA = s.

Let by the action of given force P, the particle at A be at rest. R be the normal reaction at A. Then by resolving along and perpendicular to the tangent, we have for equilibrium.

$X\cos\psi + Y\sin\psi = 0$

$\Rightarrow X\frac{dx}{ds} + Y\frac{dy}{ds} = 0$ [using (1)]

$\Rightarrow X + Y\frac{dy}{dx} = 0$ [∵ multiplying by $\frac{ds}{dx}$ both sides] and $R + Y \cos\psi = X \sin\psi$

$\Rightarrow R = X \sin\psi - Y \cos\psi$

$= X \frac{dy}{ds} - Y \frac{dx}{ds}$

$\Rightarrow R = \dfrac{X \frac{dy}{dx} - Y}{\frac{ds}{dx}}$

$= \dfrac{X \frac{dy}{dx} - Y}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$

$\left[\because \frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]$

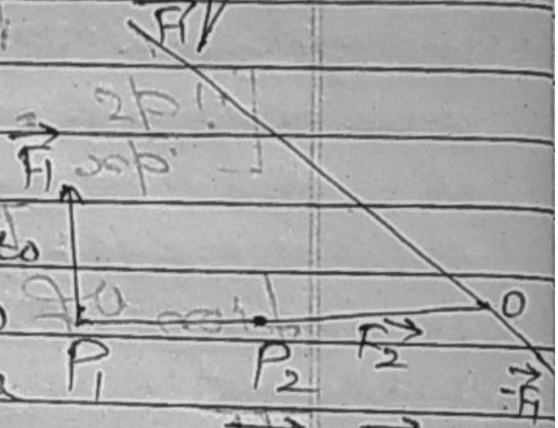
These are the position of equilibrium.

Theorem: If any system of coplanar forces acting on a rigid body then show that it is equivalent to a single force acting at an arbitrary point in the plane of the forces together with a couple.

Proof: Let us take the

forces $\vec{F_1}, \vec{F_2}, \vec{F_3}$ ---- acting at the points P_1, P_2, P_3 ---- respectively and O be any given arbitrary point themselves which not affect the effect of the given force.

The forces $-\vec{F_1}$ at O and $\vec{F_1}$ at P_1 constitute a couple of moment $\vec{OP_1} \times \vec{F_1}$ in the figure two forces $\vec{F_1}$ and $-\vec{F_1}$ at O are in equilibrium.



Hence the force $\vec{F_1}$ at P_1 is equivalent to a force $\vec{F_1}$ at O together with a couple of moment $\vec{OP_1} \times \vec{F_1}$. in the same way for $\vec{F_2}, \vec{F_3}$, we have couple of moment $\vec{OP_2} \times \vec{F_2}$, $\vec{OP_3} \times \vec{F_3}$, ---

Now all the couples can be combined in to a single resultant couple.

$$\vec{G} = \vec{OP_1} \times \vec{F_1} + \vec{OP_2} \times \vec{F_2} + \vec{OP_3} \times \vec{F_3} + ---$$

and single resultant force

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + ----$$