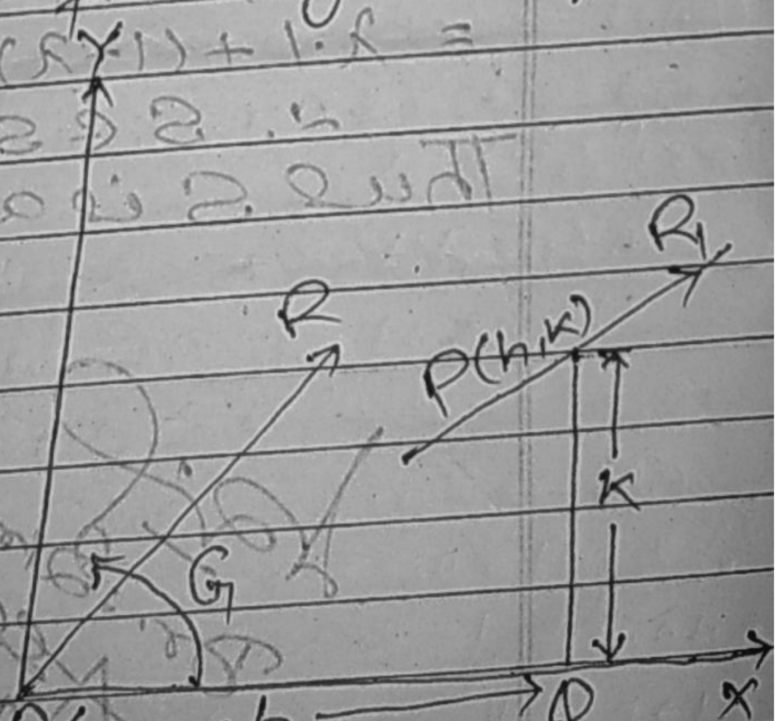
Mathematics Hons.
B.Sc. Part-II
Paper-IV
Topic: System of Coplanar forces
(Statics)

Theorem:- Find the equation to the line of action of the resultant of a system of coplanar forces.

Ans: If a system of coplanar forces acting on a rigid body then it can be reduced to a single force R in fig. Suppose R_1 is the resultant of the given system.



Let us take $P(h,k)$ be any point on resultant R_1. Then we have the moment of system about P = the moment of the resultant about $P = 0$

i.e., $G + X \cdot PQ - Y \cdot OQ = 0$.

$\Rightarrow G + X - Yh = 0$

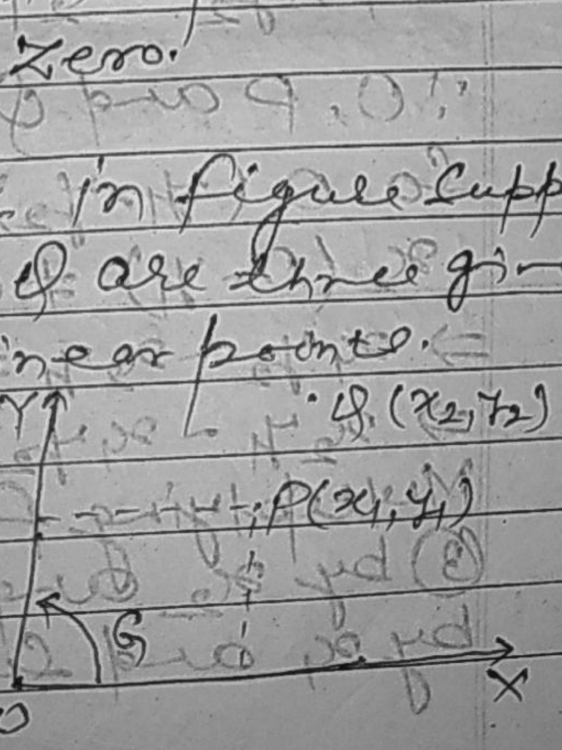
Thus the locus of (h,k) is

$$G + Xy - Yx = 0.$$

This is the required equation of the line of action of the resultant force.

Theorem: Show that a system of coplanar forces is in equilibrium if the algebraic sum of the moments about each of three given non-collinear points in the plane be zero.

Proof: In figure suppose O, P and Q are three given non-collinear points.



Here O referred as origin. Co-ordinates of P and Q are (x_1, y_1) and (x_2, y_2) respectively.

Let G_1 and G_2 be the moments of the system about $P(x_1, y_1)$ and $Q(x_2, y_2)$ respectively then we have for equilibrium.

$$\left.\begin{array}{l} G_1 = G - x_1Y + y_1X \\ \text{and } G_2 = G - x_2Y + y_2X \end{array}\right\} \quad \text{--- (1)}$$

It is given that $G = 0$, $G_1 = 0$, $G_2 = 0$ then by (1)

$$\left.\begin{array}{l} G - x_1Y + y_1X = 0 \\ \text{and } G - x_2Y + y_2X = 0 \end{array}\right\} \quad \text{--- (2)}$$

Also $G = 0$

$\therefore Xy_1 - x_1Y = 0$

and $Xy_2 - x_2Y = 0$ --- (3)

$\because$ O, P and Q are non-collinear,

So $0(y_1 - y_2) + x_1(y_2 - 0) + x_2(0 - y_1) \neq 0$.

$\Rightarrow x_1y_2 - x_2y_1 \neq 0$

$\Rightarrow x_2y_1 - x_1y_2 \neq 0$ --- (4)

Multiplying first equation of (3) by x_2 and second equation by x_1 and substract, we get

$$X\, x_2 y_1 - x_1 x_2 Y = 0$$
$$X\, x_1 y_2 - x_1 x_2 Y = 0.$$
$$X(x_2 y_1 - x_1 y_2) = 0.$$

But by (4) $x_2 y_1 - x_1 y_2 \neq 0$

$\therefore X = 0.$

In the same way multiplying first equation of (3) by y_2 and second by y_1 and substract we get

$Y = 0$ as $(x_1 y_2 - x_2 y_1) \neq 0.$

Hence we get,

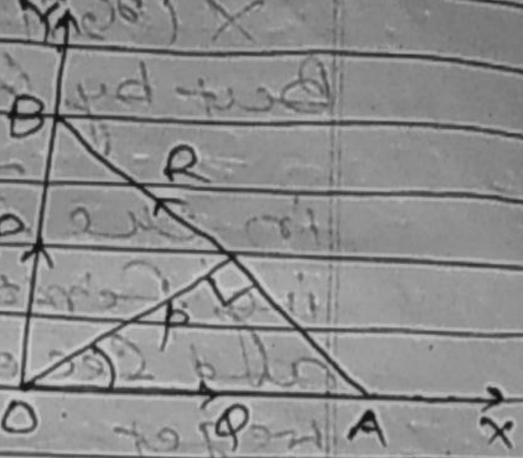
$X = 0$, $Y = 0$ and $G = 0$. These are the conditions of equilibrium.

Thus the system is in equilibrium.

Ques:- Forces P, Q and R act along the sides of the triangle formed by the lines $x = 0$, $y = 0$ and $x\cos\theta + y\sin\theta = p$ respectively, the axes being rectangular. Find the magnitude of resultant and the equation of its line of action.

Soln:- The lines of action of

the forces P, Q and R are shown in the figure. we have the magnitude of resultant



$= \sqrt{X^2+Y^2}$

$= \sqrt{(Q-R\sin\theta)^2 + (P+R\cos\theta)^2}$

[$\because X = Q - R\cos(\frac{\pi}{2}-\theta) = Q - R\sin\theta$ and

$Y = P + R\sin(\frac{\pi}{2}-\theta) = P + R\cos\theta$]

$= \sqrt{Q^2+R^2\sin^2\theta - 2QR\sin\theta + P^2 + R^2\cos^2\theta + 2PR\cos\theta}$

$= \sqrt{P^2+Q^2+R^2(\sin^2\theta+\cos^2\theta) - 2QR\sin\theta + 2PR\cos\theta}$

$= \sqrt{P^2+Q^2+R^2+2R(P\cos\theta - Q\sin\theta)}$ — (1)

This is the required magnitude of resultant. Now the algebraic sum of the moments of all the forces about origin $= G = R.p$ — (2)

we have the equation to the line of action of the resultant is

$G + yX - xY = 0$

$\Rightarrow Rp + y(Q - R\sin\theta) - x(P + R\cos\theta) = 0$

Sd/-

Dr. J [illegible]
Asst. Prof. of Maths,
Dept. of
D.K. College,
Dumraon