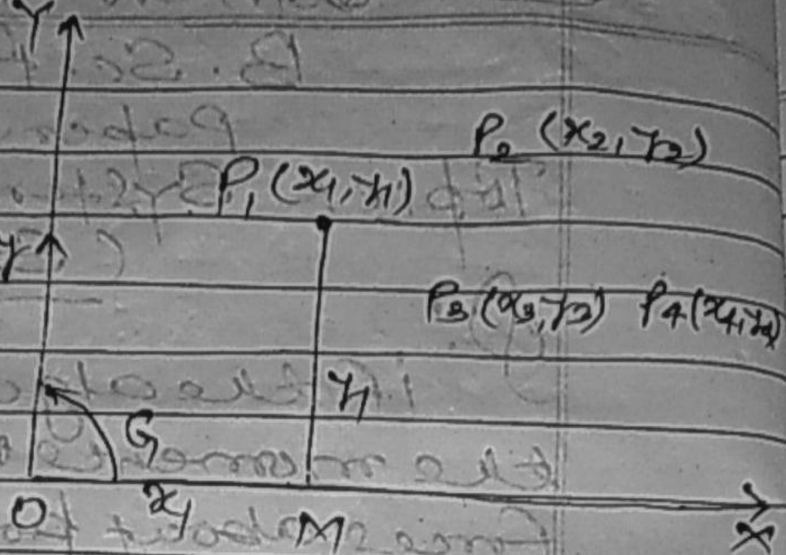
Mathematics Hons.
B. Sc. Part-II
Paper - IV
Topic: System of forces.
(Statics)

Q:
If the algebraic sums of the moments of a system of forces about points whose co-ordinates are (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) referred to Ox and OY as axes are G_1, G_2, G_3 and G_4 respectively then show that

$$\begin{vmatrix} 1 & x_1 & y_1 & G_1 \\ 1 & x_2 & y_2 & G_2 \\ 1 & x_3 & y_3 & G_3 \\ 1 & x_4 & y_4 & G_4 \end{vmatrix} = 0.$$

Solⁿ: Since the system of forces can be reduced to components X and Y along any two axes Ox and Oy and a couple G about O.

Taking the moments of the system about $P_1(x_1, y_1)$,



We have

$$G - Y \cdot OM + X \cdot P_1M - G_1 = 0.$$

$$\Rightarrow G - Yx_1 + Xy_1 - G_1 = 0. \quad \text{——(1)}$$

In the same way when we taking the moments of the system about $P_2(x_2, y_2)$, $P_3(x_3, y_3)$ and $P_4(x_4, y_4)$, we get

$$G - Yx_2 + Xy_2 - G_2 = 0. \quad \text{——(2)}$$

$$G - Yx_3 + XY_3 - G_3 = 0. \quad \text{——(3)}$$

and $G - Yx_4 + XY_4 - G_4 = 0.$ ——(4)

Now, eliminating G, Y and X from equation (1), (2), (3) and (4) we get,

$$\begin{vmatrix} 1 & x_1 & y_1 & G_1 \\ 1 & x_2 & y_2 & G_2 \\ 1 & x_3 & y_3 & G_3 \\ 1 & x_4 & y_4 & G_4 \end{vmatrix} = 0.$$

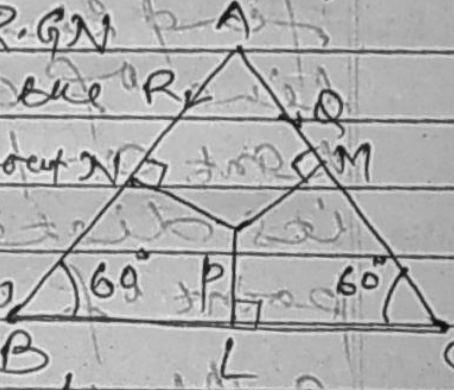
Q: Three forces P, Q, R act along the sides BC, AC, BA of an equilateral triangle ABC. If their resultant is a force parallel to BC through the centroid of the triangle, show that $Q = R = \frac{1}{2}P$.

Soln:- In equilateral triangle ABC, suppose G be centroid, GL, GM and GN be perpendiculars on BC, CA and AB respectively. Then $GL = GM = GN = p$ (say)

Taking the moments of all the forces about G, we get

$P \cdot GL - Q \cdot GM - R \cdot GN$

= Moment of the resultant about G.



$\Rightarrow P \cdot p - Q \cdot p - R \cdot p = 0$

[$\because$ resultant passes through G]

$\Rightarrow P = Q + R$ —— (1)

Now, resolving all the forces perpendicular to BC as resultant is perpendicular to BC, then

$R \sin 60 - Q \sin 60 = 0.$

$\Rightarrow R = Q$ —— (2)

So by (1)

$P = Q + Q$

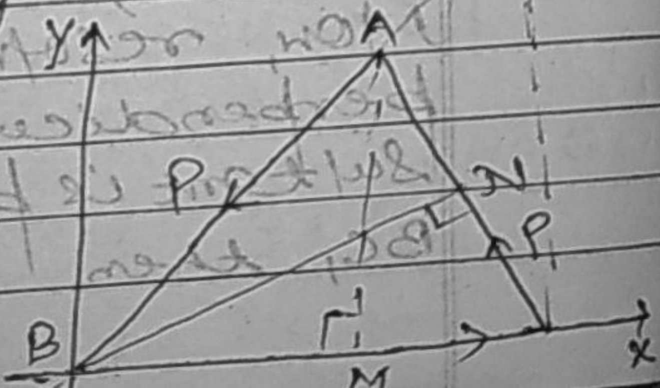
$P = 2Q$

$\therefore Q = \frac{1}{2} P.$

Hence $Q = R = \frac{1}{2} P.$

Q. Three forces, each equal to P act along the sides of the $\triangle ABC$, taken the order. Show that the resultant is $P\sqrt{1 - 8 \sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2}}$ find the equation of the line of action of the resultant and hence deduce its distance form A and its point of intersection with BC.

Sol^n:- Let us take BC as x-axis and By as y-axis. B is reffered as origin.



We have

$X = P - P\cos C - P\cos B$

$Y = P\sin C - P\sin B$

G = Sum of moments of the forces about the origin B.

$= P.BN = P.a\sin C$

$BC = a$ and $BN = a\sin C$

Since resultant $R = \sqrt{X^2+Y^2}$

$\Rightarrow R = \sqrt{(P-P\cos C-P\cos B)^2+(P\sin C-P\sin B)^2}$

$= P\sqrt{(1-\cos C-\cos B)^2+(\sin C-\sin B)^2}$

$= P\sqrt{1+\cos^2 C+\cos^2 B-2\cos C+2\cos C.\cos B-2\cos B+\sin^2 C+\sin^2 B-2\sin C.\sin B}$

$= P\sqrt{1+(\cos^2 C+\sin^2 C)+(\cos^2 B+\sin^2 B)-2\cos C-2\cos B+2(\cos C.\cos B-\sin C.\sin B)}$

$= P\sqrt{1+1+1-2\cos C-\cos B+2\cos(C+B)}$

$= P\sqrt{3-2(\cos C+\cos B)+2\cos(\pi-A)}$

$[\because A+B+C=\pi$

$\therefore C+B=\pi-A]$

$R = P\sqrt{3-2(\cos A+\cos B+\cos C)}$

$R = P\sqrt{3-2\left(1+4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}\right)}$

[$\because$ For $\triangle ABC$, $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2}$]

$\therefore R = P\sqrt{1+8 \sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2}}$

Proved.

2nd Part:

We have the equation to the line of action of the resultant is given by

$G + yX - xY = 0.$

$\Rightarrow Pa \sin C + y(P - P\cos C - P\cos B) - x(P\sin C - P\sin B) = 0.$

$\Rightarrow P[x(\sin B - \sin C) + y(1 - \cos B - \cos C) + a \sin C] = 0.$

$\Rightarrow x(\sin B - \sin C) + y(1 - \cos B - \cos C) + a \sin C = 0$ [$\because P \neq 0$]

When resultant meets at BC then $y = 0$

$\therefore x(\sin B - \sin C) + 0 + a \sin C = 0$

$\Rightarrow x = \left[\frac{-a \sin C}{\sin B - \sin C}\right]$

$\therefore x = \frac{a \sin C}{\sin C - \sin B}$

For obtained the distance of the resultant from A, we take moments of all the forces about A.

$\therefore P \times (\perp$ force A to BC$) = R \times ($distance of R from A$)$

$\Rightarrow P \cdot c \sin B = R \times ($distance of R from A$)$

$\therefore$ Distance of R from A $= \frac{P c \sin B}{R}$.