Mathematics Hons (Sub.)
Part-I
Paper-I.
Topic: LINEAR PROGRAMMING

A Linear programming (L.P) is one of the most important optimization (Maximization or minimization) techniques developed in the field of operations research.

The general L.P.P calls for optimizing (maximizing/minimizing) a linear function of variables called the 'objective function' subject to a set of linear equation inequalities called the 'constraints' or restrictions.

General Formulation of L.P.P: The general formulation of the linear programming problem may be stated as follows:

In order to find the values of n decision variables $x_1, x_2, x_3 - - - -, x_n$ to maximize or minimize the objective function

$Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$ — (1)

and also satisfy m-constraints.

$$\left.\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \; (\leq, =, \geq) b_1 \\ &a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \; (\leq, =, \geq) b_2 \\ &\cdots \\ &a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \; (\leq, =, \geq) b_m \end{aligned}\right\} \text{— (2)}$$

Where constraints may be in the form of an inequality ($\leq$, or $\geq$) or even in the form of an equation ($=$), and finally satisfy the non-negativity restrictions

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, \cdots x_n \geq 0.$

A L.P.P can be written in the form of Σ notation as maximise or minimise such that

$$Z = \sum_{i=1}^{n} c_i x_i$$

Subject to $\sum_{i=1}^{n} a_j x_i = b_j, \; j = 1, 2, 3 \ldots m$

Matrix Formulation of L.P.P: The linear programming may be stated in matrix form as follows:

Optimise: $Z = Cx$

Subject to $Ax (\leq, =, \geq) b$.

and $x \geq 0$.

where $C = \{c_1, c_2, c_3 - - - c_n\}$ is a row matrix.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$ is a column matrix

$A = [a_{ij}]$ a $m \times n$ matrix.

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$ is a column matrix

Graphical Method of solution of a L.P.P.

The graphical procedure for solving a L.P.P is in the following steps:

(1) Consider each inequalities constraints as equation.

(ii) Plot each equation on the graph as each will geometrically represent a straight line.

(iii) Shade the feasible region every point on the line will satisfy the equation of the line. If the inequality constraints corresponding to that the line is '≤' then the region below the line lying in the first quadrant due to non-negativity of variables is shaded.

For the inequality constraint with '≥' sign, the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. The common region thus obtained is called the feasible region.

(iv) Choose the convenient value of Z (say Z = 0) and of the objective function line.

(v) Pull the objective function

line until the extreme point of the feasible region. In the maximization case, this line will stop farthest from the origin and passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passing through at least one corner of the feasible region.

(vi) Read the co-ordinates of the extreme points selected in step (v) and find the maximum or minimum as the case may be of the value of z.

Dr. Birendra Pd.
Asst. Prof.
Dept. of Math.
D.K. College,
Dumraon