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Mathematics Hons. Plu.

B. Sc. Part-I

Paper-I

Topic: LINEAR PROGRA-

MMING.

Example

Old hens can be bought at Rs. 2 each and young ones Rs. 5 each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week. Each egg being worth 30 paise. A hen costs Rs. 1 per week to feed. I have only rupees 80.00 to spend for hens. How many of each kind should I buy to give a profit of more than Rs. 6 per week, assuming that I cannot house more than 20 hens.

Formulation: Let x_1 be the number of old hens and x_2 the number of young hens to be bought.

Since old hens lay 3 eggs per week and the

Young ones lay 5 eggs per week. Therefore the total eggs obtained per week

$$= 3x_1 + 5x_2$$

The total gain by the rate of 30 paise per egg

The total expenditure for x_1, x_2 hens at the rate of Rs. 1 each

Thus the total profit Z earned per week becomes = Rs. 1. $(x_1 + x_2)$

$$Z = \text{total gain} - \text{total expenditure.}$$

$$= 0.30(3x_1 + 5x_2) - (x_1 + x_2)$$

$$= 0.5x_2 - 0.1x_1$$

Since old hens can be bought at Rs. 2 each and young ones at Rs. 5 each and there are only Rs. 80 available for purchasing the hens, therefore

$$2x_1 + 5x_2 \leq 80.$$

Also, since it is not possible to house more than 20 hens at a time, therefore $x_1 + x_2 \leq 20$.

Also, since the profit is restricted to be more than Rs. 6, this means that the profit function Z is to be maximized.

Thus, there is no need
to add one more constraint
 $0.5x_2 - 0.1x_2 \geq 6$

Again, it is not possible
to purchase negative quantity
of hens, therefore,
 $x_1 \geq 0, x_2 \geq 0$

Finally, the problem becomes
find x_1 and x_2 so as to
maximize the objective function

$$Z = 0.50x_2 - 0.10x_1$$

Subject to the constraints

$$2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

Graphical Solⁿ.

Plot the
straight lines $2x_1 + 5x_2 = 80$
and $x_1 + x_2 = 20$ on the graph
and shade the feasible
region as shown in the
Graph paper.

$$2x_1 + 5x_2 = 80$$

$$\Rightarrow 2x_1 = 80 - 5x_2$$

$$x_1 = 40 - 2.5x_2$$

if $x_2 = 2$ then $x_1 = 35$

if $x_2 = 4$ then $x_1 = 30$

$x_2 = 6$ then $x_1 = 25$

and $x_2 = -2$ then $x_1 = 45$

$x_2 = -4$ then $x_1 = 50$

Table

x_1	35	30	25	45	50
x_2	2	4	6	-2	-4

$$Eq^2: x_1 + x_2 = 20$$

$$\Rightarrow x_1 = 20 - x_2$$

if $x_2 = 5$ then $x_1 = 15$

if $x_2 = 10$ then $x_1 = 10$

if $x_2 = 15$ then $x_1 = 5$

and $x_2 = -5$ then $x_1 = 25$

$x_2 = -10$ then $x_1 = 30$

$x_2 = -15$ then $x_1 = 35$

(0 + 20, 20 + 0, 0 + 0, 0 - 0)

Table

x_1	15	10	5	25	30	35
x_2	5	10	15	-5	-10	-15

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$$Z = 0.50x_1 - 0.10x_2$$

Date:

Subject to the Constraints.

$$2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

Table! $2x_1 + 5x_2 = 80$

x_1	35	30	25	45	50.
x_2	2	4	6	-2	-4

Table! $x_1 + x_2 = 20$

x_1	15	10	5	25	30	35
x_2	5	10	15	-5	-10	-15

$$2x_1 + 5x_2 = 80 \text{ --- (i) } \times 1$$

$$x_1 + x_2 = 20 \text{ --- (ii) } \times 5$$

$$\Rightarrow 2x_1 + 5x_2 = 80$$

$$5x_1 + 5x_2 = 100$$

$$\hline -3x_1 = -20$$

$$-3x_1 = -20$$

$$\therefore x_1 = \frac{20}{3} = 6.66$$

$$\Rightarrow 6.7$$

Put $x_1 = 6.7$ in eqⁿ (ii)

$$x_1 + x_2 = 20$$

$$6.7 + x_2 = 20$$

$$\therefore x_2 = 20 - 6.7$$

$$= 13.3$$

$$E(6.7, 13.3)$$

The value of Z are the vertices are

$$Z_0 = 0$$

$$Z_A = 0.50 \times 0 - 0.10 \times 16 = 0 - 1.6 = -1.6$$

$$Z_B = 0.50 \times 20 - 0.10 \times 0 = 10 - 0 = 10$$

$$Z_E = 0.50 \times 6.7 - 0.10 \times 13.3 = 2.02$$

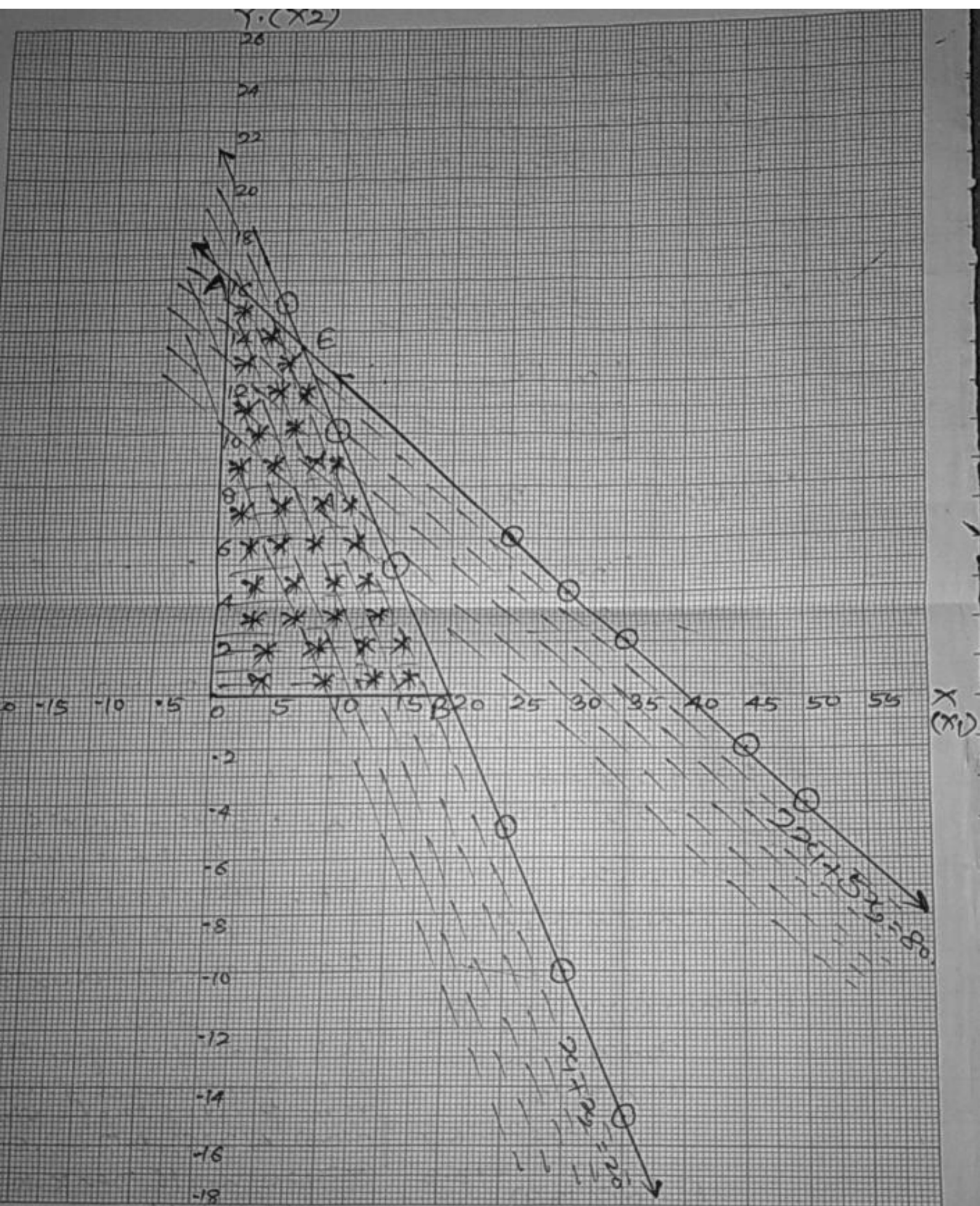
The feasible region is OBEA.

The co-ordinates of the extreme point of the feasible region

O(0,0), A(0,16), B(20,0), E(6.7,13.3)

$$\rightarrow x_1 = 20, x_2 = 16, \max Z = 10$$

Here only 16 youghens. I should by in order to get the max. profit of Rs. 10 (> 6)



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